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A NOTE ON RAMSEY PRICING – DO RAMSEY PRICES EXCEED MARGINAL COSTS?

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ABSTRACT

This paper establishes sets of sufficient conditions which guarantee that Ramsey prices exceed their corresponding marginal costs. In the literature, it is known that gross substitutabilities between the goods produced by the monopolist tend to raise their markup rates. Therefore, one might conjecture that the gross substitutabilities can guarantee that Ramsey prices exceed their marginal costs. Among others, it is established that the above conjecture holds under additional conditions.

Keywords: Profit-maximization pricing, Ramsey pricing, Ramsey taxation

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1. INTRODUCTION

In practice it is not uncommon to observe a regulated public utility that uses the profit gained from selling one product to compensate a loss from another product. For example, Chunghwa Telecom Co., Ltd. uses its profit from cellar phone services to subsidize its loss in the local phone business. Therefore, it is important to examine whether this type of cross-subsidization can be justified from an efficient point of view. In other words, this paper studies whether it is possible for Ramsey prices to fall below their corresponding marginal costs.

This paper aims to find the sufficient conditions which guarantee that Ramsey prices exceed their corresponding marginal costs (i.e., the necessary conditions for justifying the cross-subsidization). It is known that if a public utility is required to break even and its production technology has increasing returns to scale, then at least one Ramsey price must exceed its corresponding marginal cost. Furthermore, if each cross-price elasticity between commodities produced by the public utility is zero, then all Ramsey prices have been shown to exceed their corresponding marginal costs.[1] In general, however, it is an open question whether Ramsey prices exceed the marginal costs. In particular, it is conjectured that some Ramsey prices may fall below the marginal costs when some publicly-produced goods are complements.[2] Therefore, it is a real possibility for a Ramsey price to fall below its corresponding marginal cost.

It is indeed known that gross substitutabilities between the goods produced by the monopolist tend to raise their markup rates (Tirole, 1988, p. 70, for example).[3] Therefore, one might conjecture that gross substitutabilities can guarantee that Ramsey prices exceed their marginal costs. Among others, it is established that the above conjecture holds under additional conditions (see Corollary 2.2 and

[1] See Bös (1989, p. 200), for example.

[2] Refer to Bös (1989, p. 202) for the intuitive reason.

[3] What is examined by Tirole (1988) is profit-maximization pricing. However, the argument also can apply to Ramsey pricing (Please refer to Section 3 of this paper for the relationship between these two pricing problems).

Propositions 2.2–2.5).

Because the above problem is a fundamental one, it may serve some problems as a foundation. There are three such examples in the context of Ramsey taxation. First of all, Atkinson and Stern (1974, p. 123) note an important revenue effect, represented by the third term of the left-hand side of their Eq. (6), which may lead the Lagrange multiplier of the budget constraint to be less than the marginal utility of income. However, to know the impact of this revenue effect, we must first know whether each tax rate is positive when there are more than one taxed commodity (The tax rate must be positive if there is only one taxed commodity and if the public expenditure is positive). Secondly, King (1986, p. 283) notes in the second-best solution that the effective cost of the public good is equal to its production cost plus two additional terms, one of which is referred to as *Pigou term* since King argues "it measures the distortion to the aggregate willingness to pay resulting from the use of distortionary taxes to finance government expenditure." This Pigou term is represented by the third term of the right-hand side of Eq. (36) of King (1986, p. 282). It is clear that the sign of this term critically depends upon the sign of each tax rate. Thirdly, Chang (2000) aims at examining whether or not the second-best level of public good provision is lower than the first-best level, and as shown by his Propositions 1 and 2, the signs of the tax rates play essential roles.

It is well known that if the Lagrange multiplier of the budget constraint is larger than the marginal utility of income, then the profit-maximization prices have the same structure as Ramsey prices do.[4] Accordingly, Sections 2 begins by studying a simpler profit-maximization problem. In Section 2 we set out the basic model, provide sufficient conditions for prices to exceed their corresponding marginal costs, and the results are summarized in Propositions 2.1–2.5 and Corollaries 2.1–2.2.

Section 3 extends the results of Section 2 to Ramsey pricing problems (Proposition 3.1). Section 3 also addresses the question whether the Lagrange multiplier is larger than the marginal utility of income.[5] It is shown that the sufficient conditions

[4] See Bös (1989, p. 189) and Chang (1996, p. 285).

[5] This issue has received little attention. However, it serves many problems as a foundation. For example, in Bös (1989) many results are based upon the assumption that his γ is larger than zero, but less than unity, which is equivalent to the assumption that the Lagrange multiplier of the profit constraint of the public utility is positive and larger than the marginal utility of income of the

which are used to guarantee that Ramsey prices exceed the marginal costs also happen to guarantee the Lagrange multiplier to be larger than the marginal utility of income (Proposition 3.1). Section 4 concludes the present study.

2. THE SIGNS OF THE PROFIT MARGINS

The model is a standard one with one multi-product monopolist and one household. The monopolist produces n goods $x_1, \cdots, x_n$ and the price of $x_i (i = 1, \cdots, n)$ is denoted by p_i. Let $\mathbf{x} = (x_1, \cdots, x_n)'$ and $\mathbf{p} = (p_1, \cdots, p_n)'$ represent the production vector and the price vector, respectively, and "$'$" denotes the transpose. Denote the utility function of the household by $U(\mathbf{x}, z)$ where $z \in R_+$ represents other goods.

Suppose that U is strictly quasi-concave as well as twice differentiable, and hence there exists a unique solution for the utility maximization problem of the household (Mas-Colell et al., 1995, p. 68). Let $X_i(\mathbf{p}, q, y)$ $(i = 1, \cdots, n)$ be the Marshallian demand function for good x_i where q is the price of z and $y \in R_+$ is the income. Furthermore, let $Z(\mathbf{p}, q, y)$ be the Marshallian demand function for z. In addition, let $X(\mathbf{p}, q, y)$ be the vector demand function, i.e., $X(\mathbf{p}, q, y) = (X_1(\mathbf{p}, q, y), \cdots, X_n(\mathbf{p}, q, y))'$. Lastly, let $V(\mathbf{p}, q, y)$ denote the indirect utility function.

The monopolist's production technology is characterized by a total cost function $C(\mathbf{x}, q)$, which is assumed to be twice differentiable. Therefore, the monopolist's profit function is $\Pi(\mathbf{p}, q, y) = \sum_{i=1}^{n} X_i(\mathbf{p}, q, y)p_i - C(X(\mathbf{p}, q, y), q)$ and its profit-maximization problem is:

$$\max_{\mathbf{p}} \quad \Pi(\mathbf{p}, q, y) \tag{2.1}$$

where it is stressed in the notation that $\mathbf{p}$ is the control vector and the monopolist

household. Ebrill and Slutsky (1990, p. 429 and p. 440) also note that this property, although theoretically ambiguous, is essential to their results. Furthermore, in a context where the tariff revenue is used to finance a public input, Chang (1995) demonstrates that this property provides critical information about a problem examined by Feehan (1992).

takes q and y as given. Hereafter, both q and y will be omitted in the notation, except when it is necessary to mention them explicitly, since they are held constant throughout this paper. The first-order necessary conditions are

$$\frac{\partial \Pi}{\partial p_i} = X_i + \sum_{j=1}^{n}(p_j - \frac{\partial C}{\partial x_j})\frac{\partial X_j}{\partial p_i} = 0 \quad i = 1, \cdots, n \tag{2.2}$$

The remainder of this section begins with summarizing all the sufficient conditions for the profit margins to be positively established by the literature, and then proceeds to provide other new sufficient conditions. Drawing on the previous literature gives the following sufficient conditions for profit margins to be positive:

Proposition 2.1. *All profit margins are positive, if either one of the following conditions holds*:

(i) *Each cross-price elasticity between the products produced by the monopolist is zero (i.e., $\partial X_i/\partial p_j = 0 \ \forall \ i \neq j$).*

(ii) **(Spulber, 1989, Proposition 5.2.1)** *All products produced by the monopolist are compensated substitutes to each other.*[6]

(iii) **(Sandmo, 1974)** *The products produced by the monopolist are weakly separable from the other good, and the sub-utility function of* **x** *is homothetic [i.e., the utility function of the household can be written as $U(F(\mathbf{x}), z)$ and F is homothetic].*[7]

(iv) **(Sandmo, 1974)** *The products produced by the monopolist are weakly separable from the other good, and the products produced by the monopolist have the same income elasticity.*

(v) **(Deaton, 1981)** *The products produced by the monopolist are quasi-separable from the other good.*[8]

Furthermore, all the products produced by the monopolist should have a uniform markup rate if either Condition (iii) or (iv) or (v) holds.

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[6] Spulber (1989) examines the case of Ramsey pricing. However, it is easy to show that the result also holds for profit-maximization pricing.

[7] In Sandmo (1974, p. 705), either Condition (iii) or (iv) is used to guarantee that all taxed goods have the same tax rate, which, of course, is positive.

[8] For the difference between "weakly separable" and "quasi-separable," please refer to Deaton (1981, p. 1249).

The following lemma will be utilized to extend Proposition 2.1:

Lemma 2.1. *Assume that* $\mathbf{x}$ *and* z *are weakly separable. If the utility function* $U(\mathbf{x}, z)$ *is homothetic with respect to* $\mathbf{x}$ *and* z*, then* $U(\mathbf{x}, z)$ *is homothetic with respect to* $\mathbf{x}$.

Proof: See Appendix.

Proposition 2.1 leads to the following corollary:

Corollary 2.1. *Assume that the products produced by the monopolist are weakly separable from the other good. A uniform positive markup rate should apply to all the products produced by the monopolist if either one of the following conditions holds*:

(i) *The utility function is homothetic with respect to all goods, including the other good.*

(ii) *The products produced by the monopolist have a symmetric substitution matrix, i.e.,* $\partial X_i/\partial p_j = \partial X_j/\partial p_i \quad \forall i, j$.

(iii) *The marginal utility of income is constant (i.e., it does not depend upon either* $\mathbf{p}$ *or* y*).*

Proof: Part (i) follows directly from Part (iii) of Proposition 2.1 and Lemma 2.1. From footnote 9 of Silberberg (1972), it follows that if $\partial X_i/\partial p_j = \partial X_j/\partial p_i \quad \forall i, j$, then $x_1, \cdots, x_n$ have the same income elasticity. Therefore, Part (ii) follows from Part (iv) of Proposition 2.1. If the marginal utility of income is constant, then from Roy's identity and $\partial^2 V/\partial p_j \partial p_i = \partial^2 V/\partial p_i \partial p_j$ it follows that $\partial X_i/\partial p_j = \partial X_j/\partial p_i \quad \forall i, j$. Therefore, Part (iii) follows from Part (ii). Q.E.D.

Part (ii) of this corollary implies that, given the weakly separable condition, if the substitution matrix is symmetric, then the markup rates are positive even when some of the products are complements to each other.[9]

In monopolistic theory, it is not uncommon to use consumers' surplus to represent consumers' welfare, e.g., Baumol and Bradford (1970), Tirole (1988, Sec. 3.2.2), Laffont and Tirole (1994), and Armstrong et al. (1996). In this case

[9] If all of the products, including the other goods, have the same income elasticity, then each product is a normal good since the income elasticity should be strictly positive. Accordingly, from the Slutsky equation it follows that if some of the products are compensated complements to each other, then they are gross complements to each other, too. Therefore, the assumptions of Corollary 2.1 do not exclude the possibility of gross complemetarities.

the marginal utility of income is constant. Therefore, Part (iii) is of theoretical significance.

One next examines whether the profit margins are positive when the commodities are gross substitutes to each other. We are now in a position to make the following assumption:

Assumption 2.1. *Each product produced by the monopolist is a normal good.*

From the slutsky equation it follows from Assumption 2.1 that if all the products produced by the monopolist are gross substitutes to each other (i.e., $\partial X_i/\partial p_j \geq 0 \ \forall \ i \neq j$), then they are compensated substitutes to each other. Accordingly, Part (ii) of Proposition 2.1 leads to the following corollary:

Corollary 2.2. *Under Assumption 2.1, all profit margins are positive if all the products produced by the monopolist are gross substitutes to each other.*

Assumption 2.1 would be a reasonable assumption in the context of Ramsey pricing since it seems that the goods provided by public utilities are usually normal ones. However, Assumption 2.1 is not a reasonable assumption in the context of Ramsey taxation since the taxed commodities include all private goods and hence it is likely that there exists a taxed commodity which is not normal. Therefore, it is important to relax Assumption 2.1.

The following assumption is a substitute for Assumption 2.1:

Assumption 2.2. *The marginal cost of each product produced by the monopolist is constant.*

Note that the marginal cost in the Ramsey pricing literature corresponds to the pre-tax price in the Ramsey taxation literature, where it is a standard practice to assume that pre-tax prices are constant. Therefore, Assumption 2.2 is very significant, at least in the context of Ramsey Taxation.

We have the following proposition:

Proposition 2.2. *Under Assumption 2.2, all profit margins are positive if all the products produced by the monopolist are gross substitutes to each other.*

Proof: See Appendix.

Part (i) of Proposition 2.1 means that each profit margin is positive if each cross-price elasticity is zero. Therefore, each profit margin is expected to be positive if each cross-price elasticity is not large enough. We are henceforth in a position to make the following assumption:

Assumption 2.3. *$\partial X'/\partial \mathbf{p}$ is an $n \times n$ dominant diagonal matrix, i.e., $|\partial X_i/\partial p_i| > \sum_{i \neq j} |\partial X_i/\partial p_j|$.* We have the following proposition:

Proposition 2.3. *Under Assumption 2.3, all profit margins are positive if all the products produced by the monopolist are gross substitutes to each other.*[10]

Proof: Eq. (2.2) can be rewritten into

$$-\frac{\partial X'}{\partial \mathbf{p}}(\mathbf{p} - \frac{\partial C}{\partial \mathbf{x}}) = X(\mathbf{p}) \tag{2.3}$$

where $\mathbf{p} - \partial C/\partial \mathbf{x}$ is an $n \times 1$ vector whose jth element is $p_j - \partial C/\partial x_j$. Therefore, the problem studied in this paper is a special case of the linear system which has been extensively studied in Leontief's input-output analysis (Simon, 1989). Because $X_i > 0 \quad \forall i$, the result follows from Part (d) of Theorem 2 of Simon (1989). Q.E.D.

Equation (2.3) can be rewritten as follows:

$$\frac{\partial X}{\partial p_i} \cdot (\mathbf{p} - \frac{\partial C}{\partial \mathbf{x}}) = -X_i \quad \forall i$$

where $\partial X/\partial p_i$ is an $n \times 1$ vector whose jth element is $\partial X_j/\partial p_i$, and $\mathbf{a} \cdot \mathbf{b}$ is the inner product between $\mathbf{a}$ and $\mathbf{b}$. Therefore, from $X_i > 0$, $\forall i$, it follows that

$$\frac{\partial X}{\partial p_i} \cdot (\mathbf{p} - \frac{\partial C}{\partial \mathbf{x}}) < 0 \quad \forall i$$

and leads to the following proposition:

Proposition 2.4. *Suppose that the monopolist produces two goods (i.e., $n = 2$). All*

[10] It is interesting to note that $\partial X'/\partial p$ is a Metzler matrix if all the products produced by the monopolist are gross substitutes to each other. A matrix whose off-diagonal elements are all nonnegative is often called the Metzler matrix (Takayama, 1985, p. 366).

profit margins are positive if all the products produced by the monopolist are gross substitutes to each other.

Proof: $\partial X/\partial p_1$ is inside the second quadrant while $\partial X/\partial p_2$ is inside the fourth quadrant, as shown in Figure 1. If $\mathbf{p} - \partial C/\partial \mathbf{x}$ locates itself inside the second quadrant, then the inner product between $\partial X/\partial p_1$ and $\mathbf{p} - \partial C/\partial \mathbf{x}$ is positive, which is a contradiction. Similarly, it does not locate itself inside the fourth quadrant. Because at least one profit margin must be positive, it must locate itself inside the first quadrant, i.e., both profit margins are positive. Q.E.D.

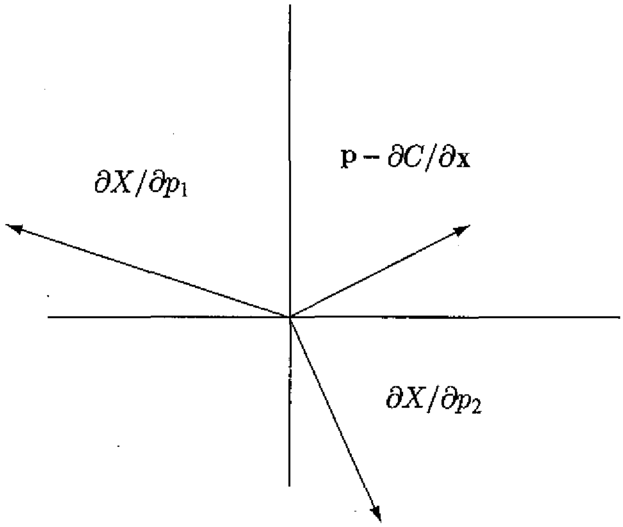

Figure 1 The Locus of the Monopolist's Profit-Maximization Solution

Quantities $x_1, \cdots, x_n$ are next chosen to be the control variables of the monopolist. This approach will make each profit margin have a simple representation.[11] Let $P_i(\mathbf{x}, q, y)$ stand for the inverse demand function of x_i given that the price of z is q and the total income is y (Note that the monopolist takes q and y as given). Again, hereafter both q and y will be omitted in the notation except when it is necessary to mention them explicitly. Furthermore, let $P(\mathbf{x}) = (P_1(\mathbf{x}), \cdots, P_n(\mathbf{x}))'$. Finally, let $\overline{\Pi}(\mathbf{x}) = R(\mathbf{x}) - C(\mathbf{x})$ where $R(\mathbf{x})$ is the total revenue function, i.e., $R(\mathbf{x}) = \mathbf{x}'P(\mathbf{x})$.

The following problem of the monopolist is next considered:

[11] This is a technique utilized by Atkinson and Stiglitz (1972) and Chang (1996). Please refer to Chang (1996, p. 285) for a discussion about this technique.

$$\max_{\mathbf{x}} \overline{\Pi}(\mathbf{x}) \tag{2.4}$$

where it is stressed in the notation that $\mathbf{x}$ is the control vector. It is straightforward to show that if $\mathbf{x}^*$ solves (2.4), then $P(\mathbf{x}^*)$ solves (2.1). This means that we can solve (2.4) instead of (2.1). It can also be shown that the first-order necessary conditions are

$$P_i - \frac{\partial C}{\partial x_i} = \sum_j (-x_j \frac{\partial P_j}{\partial x_i}) \quad i = 1, \cdots, n \tag{2.5}$$

This equation straightforwardly leads to the following proposition:

Proposition 2.5. *If the goods produced by the monopolist are substitutes in the sense that $\partial P_j / \partial x_i < 0 \ \forall \ i \neq j$, then each profit margin is positive.*

3. RAMSEY PRICING

This section utilizes a "hybrid indirect utility function" (Chang, 1996, p. 285 and p. 288) to extend the results in Section 2 to Ramsey pricing. Let

$$\overline{V}(\mathbf{x}, q, y) = V(P(\mathbf{x}, q, y), q, y) \tag{3.1}$$

Term $\overline{V}$ is the so-called "hybrid indirect utility function." Hereafter, both q and y will be omitted in the notation. Term $\overline{V}$ can be utilized to simplify the representation of Eq. (2.5). Using the definition of $\overline{V}$ and partially differentiating $\overline{V}$ with respect to x_i gives

$$\frac{\partial \overline{V}}{\partial x_i} = \sum_j \frac{\partial V}{\partial p_j} \frac{\partial P_j}{\partial x_i} = \alpha \sum_j (-x_j \frac{\partial P_j}{\partial x_i}) \tag{3.2}$$

where α is the marginal utility of income and the second equality follows from Roy's identity.

Equation (3.2) implies that Eq. (2.5) can be rewritten to be the following:

$$P_i - \frac{\partial C}{\partial x_i} = \frac{\partial \overline{V}/\partial x_i}{\alpha} \quad i = 1, \cdots, n \tag{2.5m}$$

Furthermore, from (3.2) it also follows that $\partial\overline{\Pi}/\partial x_i$ has the following representation:

$$\frac{\partial \overline{\Pi}}{\partial x_i} = (P_i - \frac{\partial C}{\partial x_i}) - \frac{\partial \overline{V}/\partial x_i}{\alpha} \tag{3.3}$$

This representation is useful for establishing the equivalence between profit-maximization pricing and Ramsey pricing.

If the quantities are chosen to be the control variables, then the Ramsey problem is

$$\max_{\mathbf{x}} \quad \overline{V}(\mathbf{x}) \qquad s.t. \quad \overline{\Pi}(\mathbf{x}) \geq 0 \tag{3.4}$$

The first-order necessary conditions include

$$\frac{\partial \overline{V}}{\partial x_i} + \lambda \frac{\partial \overline{\Pi}}{x_i} = 0 \tag{3.5}$$

where λ is the Lagrange multiplier. By using (3.3) we can rewrite (3.5) into the following form:

$$P_i - \frac{\partial C}{\partial x_i} = \frac{\lambda - \alpha}{\lambda} \frac{\partial \overline{V}/\partial x_i}{\alpha} \quad \forall i \tag{3.6}$$

This equation is obtained by letting the right-hand side of (2.5m) be multiplied by $(\lambda - \alpha)/\lambda$. Note that $(\lambda - \alpha)/\lambda$ is uniform over i.

Assume that λ is larger than α and hence $\lambda - \alpha$ is positive. In this case, a comparison between Eq. (2.5m) and Eq. (3.6) reveals that all the results obtained in Section 2 also hold for the Ramsey pricing. Thus, it is important to determine whether λ is larger than α. Actually, Eq. (3.6) leads to the following proposition:

Proposition 3.1. *Assume the technology of the monopolist has increasing returns to scale. The Lagrange multiplier of the budget constraint is larger than the marginal utility of income and each profit margin is positive if the condition listed in either one of Propositions 2.1–2.5 and Corollaries 2.1–2.2 holds.*

Proof: If the condition listed in either one of Propositions 2.1–2.5 and Corollaries 2.1–2.2 holds, then each profit margin in the profit-maximization problem is positive. From (2.5m) it follows that $\partial\overline{V}/\partial x_i$ is positive for all i. Therefore, if the Lagrange multiplier is smaller than the marginal utility of income, then it follows from (3.6) that each profit margin is negative in the Ramsey pricing problem. This contradicts the result that, in case of increasing returns to scale, at least one Ramsey price is larger than marginal cost. Q.E.D.

Proposition 3.1 shows that both the profit-maximization and Ramsey pricing problems share the same sufficient conditions. However, there is an additional sufficient condition which is specific to Ramsey pricing.[12] Drawing upon Sandmo (1974, p. 703) yields the following proposition:

Proposition 3.2. **(Sandmo, 1974)** *If the demand for the other good is completely inelastic with respect to $p_1, \cdots, p_n$, i.e., $\partial Z/\partial p_i = 0$, $\forall i$, then all markup rates are the same, and are hence positive. Furthermore, the Lagrange multiplier of the budget constraint is larger than the marginal utility of income.*

4. CONCLUDING REMARKS

To our knowledge, so far no examples where one Ramsey price actually falls below its marginal cost have been constructed in the literature, although it is noted that complementarities may produce such a perverse result. Therefore, an important mission might be to try to construct such an example.

In the context of deregulation, many regulatory issues have recently been concerned with redistributive effects (e.g., support of high-cost and/or low-income

[12] The assumption of Proposition 3.2 implies that the total revenue of the monopolist is constant, since the total revenue is equal to $y - qZ$. Therefore, the case of Proposition 3.2 is a perverse case for the profit-maximization problem.

consumers through "universal service" obligation in the telecommunications industry) and cannot be addressed within a representative consumer framework. Therefore, it is worthwhile to extend this paper to a many-person framework.[13]

Informational problems have been a central issue in the "modern" regulation literature over the last fifteen years. Therefore, needless to say, it is indeed worthwhile to further explore the conditions regarding whether the optimal prices fall below their corresponding marginal costs in the context of asymmetric information.[14]

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[13] Atkinson and Stiglitz (1976), Deaton (1979), and Deaton and Stern (1986) have already studied the conditions regarding a uniform tax rate in the context of the standard many-person Ramsey taxation. All of their conditions are also related to the weakly separable assumption.

[14] Note that the role obtained in the context of complete information also plays a role in the context of asymmetric information. For example, the right-hand first term of Eq. (24) of Laffont and Tirole (1994), which is for an asymmetric informational problem, is exactly the right-hand term of their Eq. (12), which is for the context of complete information.

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APPENDIX

The proof of Lemma 3.1:

Suppose that $U(\mathbf{x}, z) > U(\hat{\mathbf{x}}, z)$. From seeing that $U(\mathbf{x}, z)$ is homothetic with respect to **x** and z, it follows that

$$U(\gamma\mathbf{x}, \gamma z) > U(\gamma\hat{\mathbf{x}}, \gamma z) \quad \forall\gamma > 0$$

which in turn implies that

$$U(\gamma\mathbf{x}, z) > U(\gamma\hat{\mathbf{x}}, z) \quad \forall\gamma > 0$$

since **x** and z are weakly separable. Therefore, $U(\mathbf{x}, z)$ is homothetic with respect to **x**. Q.E.D.

The proof of Proposition 3.2:

It suffices to establish that if some prices fall below their corresponding marginal costs, then the profit can be strictly increased.

Let c_i stand for the marginal cost of x_i, and define

$$\Pi_i(\mathrm{p}) = (p_i - c_i)X_i(\mathrm{p}) \quad i = 1, \cdots, n$$

Suppose that some prices fall below marginal costs, say,

$$p_i < c_i, i = 1, \cdots, m, m < n; \quad p_j \geq c_j, j = m + 1, \cdots, n$$

In this case, $\Pi_i < 0 \; i = 1, \cdots, m$.

If p_i is increased to c_i $(i = 1, \cdots, m)$, then $\Pi_1 + \Pi_2 + \cdots + \Pi_m$ increases from a strictly negative number to zero. Furthermore, because $p_j \geq c_j \quad \forall j \geq m + 1$ and $x_1, \cdots, x_n$ are gross substitutes to each other, we have

$$\frac{\partial \Pi_j}{\partial p_i} = (p_j - c_j)\frac{\partial X_j}{\partial p_i} \geq 0 \quad \forall i \leq m,\ j \geq m+1$$

Accordingly, if p_i is increased to c_i $(i = 1, \cdots, m)$, then $\Pi_{m+1} + \cdots + \Pi_n$ also increases.

Remark: One innovation of this proof is that we do not utilize the first-order necessary condition to establish the result.

簡論藍氏定價是否高於邊際成本

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摘　要

本文旨在建構幾組充分條件以保證藍氏定價 (Ramsey price) 高於其邊際成本。文獻指出當產品間是粗代替品時，獨占廠商定價傾向於提高其利潤率，所以一般推測粗代替品可以保證藍氏定價高於其邊際成本。本文証明了在某些附加條件之下，上述推測可以成立。

關鍵詞：利潤最大化定價、藍氏定價、藍氏課稅

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