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計 畫 主 持 人：莊鎧溫

計畫參與人員：碩士班研究生-兼任助理人員：林兆年
碩士班研究生-兼任助理人員：李嘉銘

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一些損耗性商品的採購策略之研究

A Study of Some Replenishment Policies for Deteriorating Items

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摘要

在真實的情況中，大多數的產品會因儲存的時間過久而有損壞或腐壞的現象，例如：新鮮蔬果、食品、藥物、酒精等揮發性液體、底片和電子元件等。當此現象存在時，產品的存貨水準不僅受到需求的影響也受到退化影響。因此，當考慮此存貨管理時，應特別注意退化的特性，以免使得成本的錯估。此外，以往的文獻中甚少考慮斜坡型需求。一般新產品剛上市時，需求率常隨著時間向前移動而遞增，直到某一時點之後，需求即呈平穩狀態。本計畫將此一特性納入考慮因素以更符合一般性。

在以往的文獻中，很少考慮產品保存期限的問題，但實際上有許多商品，如食品、藥品均有保存期限的設定，一旦商品超過保存期限，商品可能仍然完好，但仍需報廢處理，因此，保存期限的問題是存貨管理必需考慮的重要課題，本計畫考慮退化性產品且有保存期限的限制，具斜坡型需求且補貨率與需求有關，建立更為實際化的模式，以供決策者做出正確判斷的參考依據。

關鍵字(詞)：退化、斜坡型需求、存貨

Abstract

In real business behavior, there are many commodity such as food, medication exist the expiry date. Therefore the existence of the expiry date has become an important research topic in production and inventory management. This study develops an inventory model for deteriorating items with a ramp-type demand and the existence of the expiry date. In the model, the finite production rate is proportional to the demand and the rate of deterioration is assumed to be constant. The optimal solution procedures for the present problems are provided. Numerical examples are presented to illustrate the models, and the sensitivity analysis of the optimal solution with respect to parameters of the systems are also carried out.

***Keyword* :** Deterioration, ramp type demand, inventory

1. Literature review

In classical inventory models, the demand rate is assumed to be a given constant. Donaldson (1977) first developed an inventory model with a linearly increasing time dependent demand rate over a finite planning horizon. Later, many research works (see, for example, Silver (1979), Goel and Aggarwal (1981), Ritchie (1984), Deb and Chaudhuri (1986), Datta and Pal (1988, 1992), Chung and Ting (1993), Kishan and Mishra (1995), Giri *et al*. (1996), Hwang (1997), Pal and Mandal (1997), Mandal and Pal (1998), and Wu *et al*. (1999)) have been devoted to incorporating a time-varying demand rate into their models for deteriorating items with or without shortages under a variety of circumstances. On the other hand, there are only a few models have been developed considering ramp-type demand in the inventory literature. This type of demand pattern is generally seen in the case of any new brand of consumer goods coming to the market. The demand rate for such items increases with time up to certain time and then ultimately stabilizes and becomes constant. It is believed that such type of demand rate is quite realistic. Mandal and Pal (1998) investigated an order-level inventory model for deteriorating items, where the demand rate as a ramp type function of time. Wu and Ouyang（2000）formulated an inventory model for deteriorating items for the prescribed cycle length and demand rate is assumed the ramp type function of time. Manna and Chaudhuri(2006) developed an EOQ model for time-dependent deteriorating items assuming the demand rate to be a ramp type function of time. In particular, the unit production cost is assumed to be inversely proportional

to the demand rate.

Deterioration is defined as decay or damage such that the item cannot be used for its original purpose. The effect of deterioration is very important in many inventory systems. Ghare and Schrader (1963) considered continuously decaying inventory for a constant demand. Covert and Philip (1973) used a variable deterioration rate of two-parameter Weibull distribution to formulate the model with assumptions of a constant demand rate. Shah and Jaiswal (1977) presented an order-level inventory model for deteriorating items with a constant rate of deterioration. Aggarwal (1978) developed an order level inventory model by correcting and modifying the error of Shah and Jaiswal's (1977) analysis to calculate the average inventory holding cost. However, all the above models are limited to the constant demand. In the literature of economic production lot size model, Dave and Patel（1981）first considered the inventory model for deteriorating items with time-varying demand. They considered a linear increasing demand rate over a finite horizon and a constant deterioration rate. Sachan（1984）extended Dave and Patel's model to allow for shortages. Chang and Dye (1999) developed an EOQ model for deteriorating items with time-varying demand and partial backlogging. Yu *et al*. (2005) developed a production-inventory model for deteriorating items with imperfect quality and partial backlogging.

In addition, a large number of researchers developed the models in the area of deteriorating inventories, but they did not consider the expiry date of items. In real business behavior, there are many commodity such as food, medication exist the expiry date. Therefore the existence of the expiry date has become an important research topic in production and inventory management. This study develops an inventory model for deteriorating items with a ramp-type demand and the existence of the expiry date. In the model, the finite production rate is proportional to the demand and the rate of deterioration is assumed to be a constant. The optimal solution procedures for the present problems are provided. Numerical examples are presented to illustrate the models, and the sensitivity analysis of the optimal solution with respect to parameters of the systems are also carried out.

2. Notation and assumptions

The mathematical model in this paper is developed on the basis of the following notations and assumptions:

Notation

C_1 ： ordering cost per order.

C_2 ： unit holding cost per unit time.

C_3 ： deterioration cost per unit of deteriorated items.

C_4 ： product expires lost cost, \$/per unit

t_1 ： point of time when inventory level is maximum

t_2 ： point of time when all inventory is consumed.

T ： production cycle.

F ： the expiry date, $0 < f < (t_2 - t_1)$

θ ： deterioration rate

μ ： specific time period of which demand is time dependent up to $t = \mu$ and then it is stabilized and constant.

$I(t)$ ： on-hand inventory at time t over $[0, t_2]$.

I_{max} ： the maximum inventory level for each ordering cycle.

OC ： ordering cost per cycle

HC ： total holding cost per cycle

DC ： total deterioration cost per cycle

LC ： product expires lost cost per cycle

T ： total cost for a production cycle

TVC ： total average cost for a production cycle.

Assumptions

1. A single item is considered and infinite planning horizon.
2. There is no replacement or repair for product expires.
3. Demand rate $R = D(t)$ is assumed to be a ramp type function of time

   $D(t)=Do[t-(t-\mu)H(t-\mu)]$,

   $D_0 > 0$ where $H(t-\mu)$ is the Heaviside's function defined as follows:

$$H(t-\mu) = \begin{cases} 1 & \text{if } t \geqslant \mu, \\ 0 & \text{if } t < \mu. \end{cases}$$

4. $P(t)=\beta\, D(t)$ is the production rate where β $(1 < \beta < 2)$ is a constant.

3. Mathematical model

Here the demand of an item is dependent on the relative size of μ. To analyze this situation, two cases may arise：

CASE1: $0 \leqq \mu < t_1$

The production starts with zero stock level at time $t = 0$. The demand rate increases with time up to time $t=\mu$. Afterwards, the demand for the product becomes constant. The production stops at time t_1 when the stock attains a level I_{max}. Due to reasons of market demand and deterioration of items, the inventory level gradually diminishes during the period $[t_1, t_2]$ and ultimately falls to zero at time $t = t_2$.

The total average cost of the system is:

$$TVC = \frac{TC}{t_1+f}$$
$$= \frac{1}{t_1+f}\{C_1 + \frac{1}{2\theta^3}\{\{-2e^{-t_1\theta}(\beta-1) + 2e^{\theta(\mu-t_1)}(\beta-1) - 2e^{-(f+t_1-t_2)}\theta\mu$$
$$+2e^{(t_2-t_1)\theta}\theta\mu + \theta\mu\{2+\theta[\mu-2(f+t_1)] + \beta(2t_1\theta - \theta\mu - 2)\}\}C_2D_0\}$$
$$-\frac{\mu[-2+2e^{-(f+t_1-t_2)\theta} + 2f\theta + (\beta-1)\theta(\mu-2t_1)]C_3D_0}{2\theta}$$
$$+\frac{(e^{(t_2-t_1-f)\theta}-1)\mu C_4 D_0}{\theta}\},$$

CASE 2: $t1 \leqq \mu < t1+f$

In this case, the variation of the inventory level with time is reflected in figure 2. The production starts with zero stock level at time $t = 0$. The demand rate increases with time up to time $t=\mu$. Afterwards, the demand for the product becomes constant. The production stops at time t_1 when the stock attains a level I_{max}. Due to reasons of market demand and deterioration of items, the inventory level gradually diminishes during the period $[t_1, t_2]$ and ultimately falls to zero at time $t = t_2$.

The total average cost of the system is:

$$TVC = \frac{TC}{t_1+f}$$
$$= \frac{1}{t_1+f}\{C_1 + \frac{1}{2\theta^3}\{e^{-(f+t_1)\theta}\{-2e^{\theta(f+\mu)} - 2e^{f\theta}(\beta-1) - 2e^{t_2\theta}\theta\mu + 2e^{(f+t_2)\theta}\theta\mu$$
$$+e^{(f+t_1)\theta}\{\beta[2+t_1\theta(t_1\theta-2)] + \theta\mu\{2+\theta[-2(f+t_1)+\mu]\}\}\}C_2D_0\}$$

$$+\frac{\{(2-2e^{-(f+t_1-t_2)\theta})\mu+\theta[t_1{}^2\beta-2(f+t_1)\mu+\mu^2]\}C_3D_0}{2\theta}$$

$$+\frac{(e^{(t_2-t_1-f)\theta}-1)\mu C_4D_0}{\theta}\},$$

4. Conclusions

In this paper, we develop an inventory model for deteriorating items with a ramp-type demand and the existence of the expiry date. In the model, the finite production rate is proportional to the demand and the rate of deterioration is assumed to be constant. In particular, the optimal solution procedures for the present problems are provided. Numerical examples are presented to illustrate the models, and the sensitivity analysis of the optimal solution with respect to parameters of the systems are also carried out. The results of sensitivity analysis indicate that the expiry date affect the optimal cycle time, production time and total average cost. The results of this study can be used by management in strategic planning.

Self-evaluation

This research corresponds to the original plan and has attained its aim. The results of this research will be submitted to an academic journal for possible publication in the near future.

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