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（商品波動與經濟活動）

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Abstract

This paper use the flexible skewed generalized t distribution (SGT) to provide an accurate characterization of the non-normal of the commodity return distributions, and analyze the time-varying scaling parameters, including those in the crude oil and gold markets. We also estimate the VaR on the basis of the GARCH-SGT model, the out-of-sample forecasting periods covers a long period, including the most unsteady period of the global financial crisis. The empirical results show that the forecasted VaR with the SGT distribution provides the most accurate out-of-sample forecasts either in the crude oil or gold markets. In the crude oil market, though all the distributions provide the correct coverage rate, the forecasted oil spot VaR with the normal distribution is appropriate only for the low confidence level, and the accuracy and performance deteriorate and lose accuracy for the higher confidence levels. In comparison, the SGT distribution provides the most accurate out-of-sample forecasts within the strict VaR confidence levels. With regard to the gold markets, the most appropriate distribution for the forecasted VaR is the SGT distribution, and the failure rates in the normal distribution and GED for the VaR are statistically higher than the specific probability of the model. Comparatively, the scaling parameters in the SGT distribution can capture the volatilities of oil and gold effectively and they show that the unexpected losses are smaller in the SGT distribution. Finally, the estimated VaR within the SGT model is significantly superior to the other distributions in the crude oil and gold markets.

Keywords: Skewed generalized t distribution; Commodity volatility; Value-at-Risk

摘要

本文使用最具包容性之一般化偏態 t 分配(skewed generalized t distribution, SGT)研究商品報酬非常態之特性，並詳細分析原油及黃金商品報酬隨時間波動之尺度參數。我們同時以 GRACH-SGT 模型估計風險值，且樣本外預測期間極長，涵蓋高度不穩定的全球金融風暴期間。實證結果顯示不管在原油或是黃金市場，SGT 分配的樣本外預測結果均比其餘分配要佳。在原油市場，雖然所有的分配在初步階段皆得以提供正確的涵蓋率，但常態分配的精準度僅在低信賴水準區間出現，且隨著信賴水準的提升，常態分配的準確度及預測績效則顯著滑落。相對的，SGT 分配則是在高信賴水準區間呈現出最佳的樣本外預測結果。在黃金市場，SGT 是表現最佳之分配，常態與 GED 分配的風險值估計失敗率皆顯著高於預期水準。再者，SGT 的尺度參數皆得以有效率的捕捉原油與黃金之報酬波動，且 SGT 分配下之未預期損失顯著較低。統整而言，在 SGT 分配下估計原油及黃金報酬的風險將顯著優於其他分配。

關鍵詞：一般化偏態 t 分配、商品波動、風險值

INTRODUCTION

Most time series are characterized by leptokurtosis and skewness, not only in financial assets (Bollerslev, 1987; Engle and Gonzales-Rivera, 1991; Ait-Sahalia and Lo, 1998; Theodossiou and Trigeorgis, 2003; Bali and Theodossiou, 2007) but also in energy assets (Solt and Swanson, 1981; Taylor, 1998; Giot and Laurent, 2003; Chan et al., 2007; Fan, 2008). Moreover, empirical evidence has shown that the conditional normal time series models are inadequate for estimating the conditional return distribution. However, relatively little work has been carried out on modeling and estimating volatilities in oil and gold assets by using non-normal distributions, for example, in oil markets. Giot and Laurent (2003), Chan et al. (2007), Fan et al. (2008), and Hung et al. (2008) comprise the limited body of work that calculate the Value-at-Risk (VaR) of commodity assets using non-normal distributions; a majority of the studies that measure the volatility of oil returns do so with normal distributions (Cabedo and Moya, 2003; Busch, 2005; Sadorsky, 2006; Sadeghu and Shavvalpour, 2006). Fan et al. (2008) pointed out that it is important to be acquainted with the characteristics of oil market risks. However, the available quantitative literatures, for example, in the gold market, are very limited. Casassus and Collin-Dufresne (2005) recently evaluated the VaR for gold, using a three factor model. This is unfortunate given the importance of oil and gold to the global economy. For participating in oil and gold markets, it is also crucial to describe the asset prices; however, no appropriate method is available for this purpose. Volatility is the principal factor for developing the economic and financial models of pricing and hedging, and estimations made under the correct specifications of the conditional distribution are more efficient. Therefore, this paper utilizes the most flexible distribution to describe the oil and gold volatilities that are characterized by leptokurtosis and skewness.

To measure market risk, the application of the VaR methodology offers comprehensive and recapitulative advantages. In practice, a key risk measure based on the VaR concept is the conditional VaR, which is the worst possible loss at a given confidence level due to adverse market movements over the next reporting period, conditional on the current portfolio volatility and market information. Mathematically, VaR is defined as a quantile of a probability distribution, used to model an underlying portfolio value or its return. It is commonly used in symmetric and normal distributions for asset returns. Portfolio VaR is often calculated on the basis of the variance-covariance approach, and returns follow the normal distribution. The most used models are the classical autoregressive conditional heteroscedasticity (ARCH)/generalized ARCH (GARCH) models, with attributes such as volatility clustering and the long-range dependence structure that exist in financial assets; moreover, these models are based on conditional Gaussian innovations (see Engle, 1982; Bollerslev, 1986). However, empirical evidence has demonstrated that the conditional normal time series models are inadequate for estimating the tail quantiles of the conditional return distributions. Substantial empirical evidence shows that the distribution of financial returns is typically skewed, peaked around the mean (leptokurtic) and characterized by fat tails. Bollerslev, Engle, and Nelson (1994) proposed that the leptokurtosis is reduced, but not eliminated, when returns are standardized using time-varying estimates for the means and variances. This prompts the gradual adoption of models with heavy-tailed innovations in risk modeling. Many extensions of the classical GARCH models with heavy-tailed innovations have been proposed.

Student's t, generalized error distribution (GED), and a mixture of two normal distributions are frequently used for describing the non-normal characteristics in the VaR literature. With regard to the commodity markets, Giot and Laurent (2003) compared the performance of the RiskMetrics, skewed Student asymmetric power GARCH (APGARCH), and skewed Student ARCH models for several commodities. They found that the skewed Student ARCH model delivered excellent results and was relatively easy to use. Chan et al. (2007) considered a GARCH model with heavy-tailed innovations and characterized the limiting distribution of an estimator of the conditional VaR, which corresponds to the external quantile of the conditional distribution of the GARCH process. Fan et al. (2008) estimated the VaR of the returns in West Texas Intermediate (WTI) and Brent crude oil spot markets using a GED-GARCH model. They found this approach to be more realistic and comprehensive than the commonly used standard normal distribution-based VaR model, and also more effective than the well-recognized historical simulation with autoregressive moving average (ARMA) forecasts. Hung et al. (2008) investigated the fat-tailed innovation process on the VaR estimates, and the empirical results showed that the GARCH-HT model is quite accurate and efficient in estimating the VaR for energy commodities.

However, because such distributions partially deal with the issues of leptokurtosis and skewness, they cannot fully correct the measurement bias in risk problems (Bali and Theodossiou, 2007). The skew generalized t (SGT) distribution, introduced by Theodossiou (1998), is a skewed extension of the generalized-t distribution, originally proposed by McDonald and Newey (1988). The SGT is a distribution that allows for a very diverse level of skewness and kurtosis, and it has been used to model the unconditional distribution of daily returns for a variety of financial assets (Theodossiou, 1998; Harris and Kucukozmen, 2001). Furthermore, the SGT nests several well-known distributions such as the generalized t (GT) of MacDonald and Newey (1988); the skewed t (ST) of Hansen (1994); the skewed generalized error distribution (SGED) of Theodossiou (2001); and the normal, Laplace, uniform, GED, and student t distributions. Harris et al. (2004) further found that a conditional SGT distribution offers a substantial improvement in the fit of the GARCH model for stock index assets. Bali and Theodossiou (2007) proposed a conditional technique for estimating the VaR and expected shortfall measures on the basis of the SGT distribution in the S&P 500 index returns. They found that GARCH-type models with the SGT distribution are much superior to the conditional normal distribution for all GARCH specifications and all probability levels. Bali et al. (2008) also used the SGT distribution with time-varying parameters to provide an accurate characterization of the tail of the standardized equity return distributions. To fill in the gap in the inadequate research in which the SGT distribution in non-normal commodity returns has been employed, we use the GARCH-SGT model to model the commodity volatilities. The analytical and empirical results in this paper could provide better approximations of reality.

The remainder of this paper is organized as follows. Section 2 describes the motivation behind focusing on the crude oil and gold markets. Section 3 presents the methodologies of the GARCH-SGT models and the measurement of the VaR. Section 4 compares the out-of-sample empirical results of the SGT, and the normal distribution and GED. Section 5 concludes the paper.

IMPORTANCE OF CRUDE OIL AND GOLD

Oil is one of the most important commodities, and almost everything tangible that we physically move burns oil in the process. One of the characteristics of the oil market prices is volatility, which is both high and variable over time. In general, oil prices have become more volatile since 1986 (Plourde and Watkins, 1998; Lynch, 2002; Regnier, 2007), and this volatility has a significant impact on the global economy (Lee et al., 1995; Ferderer, 1996; Sadorsky, 1999, 2006). US oil prices have been heavily regulated through production or price control measures throughout much of the twentieth century. With the exception of the occasional jump in late 1990, crude oil prices have risen progressively since the later part of 2001. In January 2008, oil prices unprecedentedly surpassed \$100 a barrel, the first of many price milestones to be passed in the course of the year. In July 2008, oil prices peaked at \$147.30 a barrel. In the second half of 2008, the prices of most commodities fell dramatically in anticipation of diminished demand owing to the recent global recession. In fact, these high prices resulted in a dramatic drop in demand and prices fell below \$35 a barrel at the end of 2008. It is believed that high prices will cause genuine economic damage, resulting in the threat of stagflation and a reversal of globalization. In July 2009, the president of the Organization of Petroleum Exporting Countries (OPEC), Jose Maria Botelho de Vasconcelos, remarked that a crude oil price of \$68–\$71 a barrel was optimal for a stable industry. The oil market was very fragile, and crude prices were susceptible to huge fluctuations caused by minor events. Factors such as high demand, low supply, strategies adopted by OPEC, environmental regulations, hedge fund actions, and violence in the Middle East have all stimulated prices. A traditional demand-based framework was unable to explain the marked deterioration in the commodity and oil prices (Chaudhuri, 2001). Jalali-Naini and Manesh (2006) also pointed out that high volatility is a very promising characteristic for testing volatility models.

Of all the precious metals, gold is the most popular as an investment. Investors generally buy gold as a hedge or safeguard against any economic, political, social, or currency-based crises. History has shown that in adverse periods, investors tried to preserve their assets by investing in precious metals, most notably gold and silver. Since April 2001, the gold price has more than tripled in value against the US dollar, prompting speculation that this long secular bear market (or the Great Commodities Depression) has ended and a bull market has reemerged. In March 2008, the gold price increased above \$1,000. A number of studies have reported on the relationship between gold and macroeconomic variables (Sherman, 1983; Baker and Van-Tassel, 1985; Kaufmann and Winters, 1989; Sjaastad and Scacciavillani, 1996; Taylor, 1998; Christie-David et al., 2000; Cai et al., 2001; Tully and Lucey, 2006). These studies confirmed that macroeconomic variables such as the exchange rate of dollar, stock index, interest rate, consumer price index (CPI), and unemployment rate influence gold returns. In contrast, Lawrence (2003) argued that no significant correlations exist between gold returns and changes in certain macroeconomic variables.

To address the ambiguous empirical results in measuring the VaR within oil and gold markets, this paper provides a comprehensive analysis using the flexible SGT distribution for modeling the volatilities. This paper extends the existing research in oil and gold markets in four important ways. First, we calculate the VaR on the basis of the SGT—a distribution that allows for a very diverse level of skewness and kurtosis—for modeling the distribution of commodity returns. The normal distribution and GED are the comparable models used to assess the robustness of the SGT distribution. Second, considering the behavior of highly volatile oil and gold assets, we employ the GARCH models for estimating the time-varying conditional variance of returns. Third, we analyze the time-varying scaling parameters of crude oil and gold assets. It will be easy to observe why traditional distributions are not appropriate for estimating volatilities and forecasting the VaR. Fourth, this paper investigates the volatility in the prices—both spot and futures—of oil and gold assets. This paper also analyzes the performance of out-of-sample forecasting for a long period, covering both stable and high-fluctuation periods, including the period of the current global financial crisis. The VaR in the SGT distribution is significantly superior to other distributions.

METHODOLOGY

GARCH(1,1) Model with Skewed Generalized T Distribution (GARCH-SGT)

This paper investigates GARCH(1,1) model in computing the conditional means and conditional variances for conditional VaR analysis. The GARCH(1,1) model proposed by Bollerslev (1986) is as follows:

$$r_t = \mu_t + \varepsilon_t, \quad \varepsilon_t \sim (0, h_t) \tag{1}$$

$$h_t = \beta_0 + \beta_1 h_{t-1} z_{t-1}^2 + \beta_2 h_{t-1} \tag{2}$$

where $\beta_0 > 0$, $\beta_1 > 0$, $\beta_2 > 0$ and $\beta_1 + \beta_2 \leq 1$. In the equations, μ_t and h_t are the conditional mean and conditional standard variance of returns r_t based on the information set Ω_{t-1} up to time t-1. The standardized error term is $z_t = \varepsilon_t / \sqrt{h_t}$.

Considering the non-normal characteristics of energy assets, the conventional GARCH model with normal distribution is fail to capture the behavior of high-volatility of oil and golf assets. SGT distribution, advanced by Theodossiou (1998), is displaced for well-describing the distribution of assets returns exhibiting skewness and leptokurtosis. The probability density function for the SGT distribution can be represented as follows:

$$f(z_t | n, \kappa, \lambda) = C\left(1 + \frac{|z_t + \delta|^\kappa}{((n+1)|\kappa)(1 + \mathrm{sign}(z_t + \delta)\lambda)^\kappa \theta^\kappa}\right)^{-\frac{n+1}{\kappa}} \tag{3}$$

where $C = 0.5\kappa\left(\frac{n+1}{\kappa}\right)^{-\frac{1}{\kappa}} B\left(\frac{n}{k}, \frac{1}{k}\right)^{-1} \theta^{-1}$, $\theta = \frac{1}{\sqrt{g - \rho^2}}$, $\delta = \rho\theta$,

$$\rho = 2\lambda B\left(\frac{n}{\kappa}, \frac{1}{\kappa}\right)^{-1} \left(\frac{n+1}{\kappa}\right)^{\frac{1}{\kappa}} B\left(\frac{n-1}{\kappa}, \frac{2}{\kappa}\right)$$

$$g = (1 + 3\lambda^2) B\left(\frac{n}{\kappa}, \frac{1}{\kappa}\right)^{-1} \left(\frac{n+1}{\kappa}\right)^{\frac{2}{\kappa}} B\left(\frac{n-2}{\kappa}, \frac{3}{\kappa}\right)$$

where λ is a skewness parameter, "sign" is the sign function, $B(\cdot)$ is the beta function, and δ is the Pearson's skewness and mode of $f(z_t)$. The scaling parameters n, κ and λ obey the following constraints: $n > 2$, $\kappa > 0$ and $-1 < \lambda < 1$. The skew parameter λ controls the rate of descent of the density around the mode of z. In the case of positive skewness ($\lambda > 0$), the density function is skewed to the right. In contrary, the density function is skew to the left with the negative skewness ($\lambda < 0$). The parameter n and κ control the tail and height of the density. Smaller values of κ and n result in larger values for the kurtosis (i.e. more leptokurtosis p.d.f.s) and vice versa. The parameter κ (>0) determines the (fat) tail and height or shape (degree of leptokurtosis) of the distributions (eq., normal for $\kappa = 2$ and Laplace for $\kappa = 1$; thinner tail than normal for $\kappa > 2$ vs. thicker tail than normal for $\kappa < 2$). The parameter n has the degree of freedom interpretation in the case $\lambda = 0$ and $\kappa = 2$. Moreover, larger positive values of λ result in larger positive values for both skewness and kurtosis (Theodossiou, 1998).

The SGT distribution nests several well-known distributions (see Table 1). Specifically, it gives for $\lambda = 0$, McDonald's and Newey's(1988) GT distribution; for $\kappa = 2$, Hansen(1994)'s skewed student's *t* distribution; for $\lambda = 0$ and $\kappa = 2$, the student's *t* distribution; for $\lambda = 0$ and $n = \infty$, the Subbotin(1923)'s power exponential distribution; for $\lambda = 0$, $\kappa = 1$ and $n = \infty$, the Laplace distribution; for $\lambda = 0$, $\kappa = 2$ and $n = 1$, the Cauchy distribution; for $\lambda = 0$, $\kappa = 2$, and $n = \infty$, the normal distribution; and for $\lambda = 0$, $\kappa = \infty$, and $n = \infty$, the uniform distribution. Furthermore, the conditional version of SGT for $\kappa = 2$ nests the conditional skewed t distribution of Jondeau and Rockkinger (2003).

The log-likelihood function of the GARCH-SGT model can be written as:

$$\underset{SGT}{\mathrm{LogL}} = \sum_{t=1}^{T} \ln f(z_t | n, \kappa, \lambda) \tag{4}$$

Measurement and Performance in VaR

Definition and estimation

A VaR model measures market risk for a portfolio of financial assets and measures the potential loss that a portfolio could lose over a given period of time. The manager may be interested in making a statement of the following form: "We are p percent certain that we will lose more than υ dollars in the next N days." The variable υ_t is the VaR of the portfolio. Mathematically, the function can be expressed as:

$$p = \int_{-\infty}^{\upsilon_t} f_t(r)dr, \tag{5}$$

where $f_t(r)$ represents the probability density function of return r_t, the change in the value of a portfolio over a certain horizon N days. The one-day-ahead VaR based on the GARCH-SGT can be calculated as:

$$\mathrm{VaR}_{t+1}^{SGT} = f_\alpha(z_t; n, \kappa, \lambda) \cdot \sqrt{h_t} - E(r) \tag{6}$$

where $f_\alpha(z_t; n, \kappa, \lambda)$ denotes the left quantile at α for SGT distribution[1] with scaling parameters n, κ and λ. The h_t is the conditional variance of the GARCH model.

Test of correct conditional coverage

A "failure" is defined as an outcome $r_t < \upsilon_t$. Intuitively, a "good" VaR estimators υ_t would be such that $\Pr(r_t < \hat{\upsilon}_t)$ is close to p. The indicator variable is set as followed,

$$I_t = \begin{cases} 1, & \text{if} \quad r_t < \upsilon_t \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

The stochastic process $\{I_t\}$ is called the failure process. The VaR forecasts are said to be efficient of they display correct conditional coverage, that is, $E(I_{t|t-1}) = p \ \forall \ t$. Kupeic (1995) develops a test for correct unconditional coverage in the likelihood ratio (LR) framework. The likelihood ratio statistics is as follows:

$$LR_{uc} = -2\log\left[\frac{p^{n_1}(1-p)^{n_0}}{\hat{\pi}^{n_1}(1-\hat{\pi}^{n_0}}\right] \sim \chi^2_{(1)} \tag{8}$$

where p is the tolerance level where VaR measures are estimated, n_1 (n_0) is the number of 1 (0) in the indicator series, and $\hat{\pi} = n_1/(n_1 + n_0)$, the MLE of p. The null hypothesis of the failure probability p is tested against the alternative hypothesis that the failure probability is different from p.

Evaluation using regulatory loss function

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[1] The quantiles of the SGT distribution with various combinations of shape parameters are calculated with numerical integration or bootstrapping technique.

The loss function evaluation method proposed based on assigning to VaR estimates a numerical score that reflects specific regulatory concerns. It provides a measure of relative performance that can be used to monitor the performance of VaR estimates. Two regulatory loss functions proposed by Lopez (1998) are described below.

(1) Binary loss function

If the predicted VaR is not able to cover the realized loss, this is termed a 'violation'. A binary loss function is merely the reflection of the LR test of unconditional coverage test and gives a penalty of one to each exception of the VaR. Namely,

$$L_{t+1} = \begin{cases} 1, & \text{if} \quad r_{t+1} < \upsilon_{t+1} \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

If a VaR model truly provides the level of coverage defined by its confidence level, then the average binary loss function over the full sample will equal p for the (1-p) th percentile VaR

(2) Quadratic loss function

The quadratic loss function of Lopez (1998) penalizes violations differently from the binary loss function, and pays attention to the magnitude of the violation. That is,

$$L_{t+1} = \begin{cases} 1 + (r_{t+1} - \upsilon_{t+1})^2 , & \text{if } r_{t+1} < \upsilon_{t+1} \\ 0 & , \text{if } r_{t+1} \geq \upsilon_{t+1} \end{cases} \tag{10}$$

The quadratic term ensures that large violations are penalized more than the small violations which provide a more powerful measure of model accuracy than the binary loss function.

EMPIRICAL RESULTS

Out-of-sample forecasting performance: Crude oil markets

To assess the forecasting performance with alternative distributions, we first make estimates on the basis of daily returns for two years, after which the estimation period is continuously rolled forward by adding the most recent day and excluding the oldest. Following this process, the out-of-sample VaRs are calculated for the next 1,800 days (from January 2002 to March 2009); the results are illustrated in Figure 2. The forecasting performance can be analyzed in terms of the integrity of the results for the long forecasting period, which includes both stable and high volatility periods, especially through the global financial crisis period from 2007. Tables 3 and 4 list the out-of-sample forecasting results for crude oil and gold in this paper; Panels A and B show the spot and futures prices, respectively.

We first discuss the results of the crude oil spot market. All the statistics are not significant in the correct unconditional coverage test (LRuc), thus indicating that the estimated failure probability is statistically consistent with the specified probability of the model. We then compare the unexpected loss (UL) and the average quadratic loss function (AQLF[2]). For the low confidence level (95% VaR), the normal distribution yields the highest VaR estimates and the lowest failure rates in the AQLF and UL. In comparison, the failure rates in the GED and SGT distribution are higher for the low confidence level of 95% VaR. Although these distributions provide a correct coverage rate, the SGT distribution and GED have lower accuracy than normal distribution.

[2] AQLF is the final standard when we select the best model, because large violations are penalized more than the small violations, and it provides a more powerful measure of model accuracy than other standards.

However, the results are completely different for the high confidence levels (99% and 99.5%). The most correct VaR estimates are with the SGT distribution, whereas the accuracy and performance with the GED and normal distribution deteriorate and lose accuracy. A comparison of the AQLF values reveals that the lowest value is 0.0597 and 0.0362 for 99% and 99.5%, respectively, with the SGT distribution; 0.0699 and 0.0445, respectively, with the GED; and 0.0785 and 0.0542, respectively, with the normal distribution. It is obvious that the SGT distribution provides the most accurate out-of-sample forecasts within the strict VaR confidence levels. With regard to the crude oil futures prices, the SGT distribution shows the best performance in any confidence level. Part B in Table 3 shows that although all the estimated failure rates are statistically consistent with the specified probability of the model, the AQLF and UL are the lowest with the SGT distribution. The best (or lowest) AQLF for 95%, 99%, and 99.5% VaR is 0.0185, 0.0283, and 0.0119, respectively, with the SGT distribution, and the inferior AQLF is with the GED; further, the worst (or highest) AQLF is 0.1878, 0.0482, and 0.0293, respectively, with the normal distribution. In sum, the skewness and leptokurtosis result in the improper VaR estimates with the normal distribution.

Next, we illustrate the time-varying scaling parameters in Figure 3 for analyzing the superiority of the SGT distribution in the crude oil spot prices[3]. Two lines are drawn: the solid line indicates the values as on August 9, 2007, and the dotted line indicates the values as in September 2008. The former date indicates the beginning of the global financial crisis, which resulted in a liquidity crisis that prompted a substantial injection of capital into the financial markets by the United States Federal Reserve, Bank of England, and the European Central Bank. In the latter date, September 2008, the crisis deepened, as stock markets worldwide crashed and entered a period of high volatility, and a large number of banks, mortgage lenders, and insurance companies failed in the subsequent weeks. In the part in the figure indicating the GED, we can see that the fat-tail parameter (κ) is below 2 in the forecasting period, indicating that the fat-tail exists in the crude oil spot prices. However, the fluctuation of the parameter κ is not large, except in the global financial crisis period when it is comparatively low. In comparison, an observation of the scaling parameters of the SGT distribution in Part B of Figure 3 shows that the skewness parameter λ is smooth around 0, thus indicating that the skewness is not very important. However, two kurtosis parameters (κ and n) perform differently in the forecasting period. The first parameter, κ, is very smooth and the average value is close to that of the normal distribution (i.e., 2), indicating no peakness for the empirical distribution. The second parameter, n, on the other hand, is very volatile, especially in the beginning of the global financial crisis. By definition, the smaller values of κ and n result in larger values for the kurtosis, and vice versa, and the SGT distribution is close to the normal distribution while $\lambda = 0$, $\kappa = 2$, and $n = \infty$. We can therefore say that the normal distribution is appropriate only for the starting year of the global financial crisis and the fat-tail distribution is more appropriate for other periods. To eliminate the extremely high value for parameter n, we redraw the scaling parameters in Part C in Figure 3. It can be clearly seen that the fluctuation in parameter n and the estimated values of the parameter were not very large except in mid-2003 and early 2004, indicating significant fat tails for the empirical distribution of standardized returns. This is why the forecasting performance with the normal distribution was not better than that with the alternative distributions.

Out-of-sample forecasting performance: Gold markets

[3] The results are shown in terms of crude oil futures prices.

Next, we analyze the gold spot market, for all the confidence levels. The estimated failure rates with the SGT distribution are the only ones that pass the coverage rate test, LRuc. However, the estimated failure rates with either the normal distribution or GED are rejected in the LRuc tests. The phenomenon represents the failure rates in the normal distribution and GED, because the VaRs are statistically higher than the specific probability of the model. For example, in the normal distribution, the estimated failure rates are 0.0605, 0.0250, and 0.0183, which are statistically much higher than the specific probabilities of 0.05, 0.01, and 0.005. The same results appear in the GED, where the estimated failure rates are 0.0622, 0.0177, and 0.0083, which significantly exceed the specific probabilities of 0.05, 0.01, and 0.005. Identical results are shown in the AQLF and UL: the values are the lowest with the SGT distribution irrespective of the confidence level (95%, 99%, and 99.5%). With regard to the futures market, though the estimated failure rates either with the normal distribution or GED are fine in 95% VaR, the failure rates are significantly biased in higher confidence levels (99% and 99.5% VaR). Let us take the normal distribution, for example. The estimated failure rates are 0.0216 and 0.0161 for 99% and 99.5% confidence levels, respectively; these values significantly exceed the specific probabilities of 0.01 and 0.005, respectively. In comparison, the estimated failure rates with the SGT distribution pass the coverage rate test in all the confidence levels. Similar results can be found in the AQLF and UL, and the values with both the normal distribution and GED are relatively higher than those with the SGT distribution.

We illustrate the time-varying scaling parameters in Figure 4. In the part indicating the GED in the figure, we see that the fat-tail parameter (κ) fluctuates around 2 prior to August 2007; moreover, there is a clear decline in the parameter in the global financial crisis period. Specifically, in the GED, the fat-tail is more apparent in the financial crisis period, but not so much in the other periods. In comparison, the scaling parameters of the SGT distribution in Part B of Figure 4 show that the skewness parameter λ is smooth around 0, indicating that the skewness is not very significant. However, two kurtosis parameters (κ and n) perform differently in the forecasting period. The first parameter, κ, is quite stable in the whole period and the average value is around 2, whereas the second parameter, n, is relatively volatile as compared to parameter κ. Except for the beginning of 2003 and mid-2007, the value of parameter n is low, indicating that the kurtosis exists significantly. In comparison, the scaling parameters in the SGT distribution can appropriately capture the volatility of gold and they show that the unexpected losses are smaller in the SGT distribution. Figure 5 shows the results of the forecasted VaR. Focusing on the latest period of the global financial crisis, we can easily observe that the forecasted VaR with the normal distribution and GED cannot capture the situation of high volatility, and the forecasted loss increases. However, the forecasted VaR with the SGT distribution is apparently different: the forecasted errors are relatively much smaller than the alternative distributions.

In sum, the forecasted VaR with the SGT distribution provides the most accurate out-of-sample forecasts either in the crude oil or gold markets. In the crude oil market, though all the distributions provide the correct coverage rate, the forecasted oil spot VaR with the normal distribution is only appropriate for the low confidence level, and the accuracy and performance distributions deteriorate and lose accuracy for the higher confidence levels. In comparison, the SGT distribution provides the most accurate out-of-sample forecasts within the strict VaR confidence levels. With regard to the gold market, the most appropriate distribution for the forecasted VaR is with the SGT distribution, and the failure rates in the normal distribution and GED for the VaR are statistically higher than the specific probability of the model. Therefore, the precise forecasting is the most important reason for adopting the SGT distribution.

CONCLUSION

This paper provides a comprehensive analysis using the flexible SGT distribution for modeling commodity volatilities and analyzing the time-varying scaling parameters, including those in the crude oil and gold markets. It also estimates the VaR within the framework of the GARCH-SGT model. The out-of-sample forecasting period covers a long period, including the most unsteady period of the global financial crisis. The empirical results show that the forecasted VaR with the SGT distribution provides the most accurate out-of-sample forecasts either in the crude oil or gold markets. In the crude oil market, though all the distributions provide the correct coverage rate, the forecasted oil spot VaR with the normal distribution is appropriate only for the low confidence level, and the accuracy and performance distributions deteriorate and lose accuracy for the higher confidence levels. In comparison, the SGT distribution provides the most accurate out-of-sample forecasts within the strict VaR confidence levels. Further, the time-varying parameters in the SGT distribution show that two kurtosis parameters (κ and n) perform differently in the forecasting period. The peakness parameter is close to that of the normal distribution (i.e., 2), indicating no peakness for the empirical distribution. However, the fat-tail characteristic significantly exists for the empirical distribution of returns. Focusing on the period of the current global financial crisis, we see that the estimation results of the scaling parameters in the GED and SGT distribution are totally different: the SGT distribution allows a very diverse level of skewness and kurtosis, and can capture the volatility more effectively. This is why the forecasting performance with the normal distribution and GED is not better than with the SGT distribution.

With regard to the gold markets, the most appropriate distribution for the forecasted VaR is the SGT distribution, and the failure rates in the normal distribution and GED for the VaR are statistically higher than the specific probability of the model. Moreover, the time-varying scaling parameters are similar to crude oil returns. The skewness parameter is close to 0, indicating that the skewness is not very significant; the peakness parameter is close to that of the normal distribution, indicating no peakness for the empirical distribution; and finally, the fat-tail parameter is small, indicating that the kurtosis significantly exists. Comparatively, the scaling parameters in the SGT distribution can capture the volatilities of gold effectively and they show that the unexpected losses are smaller in the SGT distribution. We focused on the latest period of the global financial crisis and found that the forecasted VaR with the normal distribution and GED are biased, whereas the SGT distribution can model the high volatility well. Finally, the estimated VaR within the SGT model is significantly superior to the other distributions in the crude oil and gold markets.

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Table 1. The Special Cases of SGT distributions

	λ	κ	n	Notes:	
Skew generalized t (SGT)	Free	Free	Free	$\lambda > 0$	skew to the right
Skew t (ST)	Free	2	Free	$\lambda < 0$	skew to the left
Skew GED (SGED)	Free	Free	∞		
Skew Normal	Free	2	∞	$\kappa > 2$	thinner tail than normal
Skew Laplace	Free	1	∞	$\kappa < 2$	thicker tail than normal
General t (GT)	0	Free	Free		
Student t	0	2	Free		
GED	0	Free	∞		
Normal	0	2	∞		
Laplace	0	1	∞		
Uniform	0	∞	∞		

Table 2. Descriptive Statistics

	Mean	S.D.	Skewness	Excess Kurtosis	J-B test
Part A. Crude oil					
Spot price	49.9529	26.3458	1.2846**	1.2981**	795.8244**
return	0.011	2.661	-0.2766**	4.2950**	1801.1006**
Futures price	49.9703	26.3638	1.2793**	1.2867**	787.7146**
return	0.0276	2.6552	-0.2082**	4.0639**	1602.8249**
Part B. Gold					
Spot price	484.8145	208.9687	0.8346**	-0.5371**	295.0343**
return	0.0518	1.8027	-0.2259**	23.0336**	50930.0510**
Futures price	486.2587	209.9169	0.8247**	-0.5695**	292.1880**
return	0.0511	1.2118	0.0956*	5.0259**	2427.3922**

Notes: J-B test is Jarque-Bera normality test. **and * represent significance under 1% and 10% level.

Table 3. Out-of-sample performance of alternative distributions for crude oil

Part A. Spot

		Mean	S.D.	LRuc	ABLF	AQLF	UL
Normal	95% VaR	-3.9992	1.7264	0.0464	0.0511	**0.2385**	**-0.0633**
	99% VaR	-5.7055	2.4509	2.4527	0.0138	0.0785	-0.0185
	99.5% VaR	-6.3302	2.7162	1.5697	0.0072	0.0542	-0.0126
GED	95% VaR	-3.9632	1.6979	0.0000	0.0500	0.2626	-0.0667
	99% VaR	-6.0390	2.5028	0.2166	0.0111	0.0699	-0.0172
	99.5% VaR	-6.8477	2.8146	1.5697	0.0072	0.0445	-0.0110
SGT	95% VaR	-3.9120	1.6634	0.8054	0.0546	0.2616	-0.0681
	99% VaR	-6.1914	2.4670	0.0485	0.0094	**0.0597**	**-0.0139**
	99.5% VaR	-7.2397	2.8140	0.1169	0.0055	**0.0362**	**-0.0077**

Part B. Futures

		Mean	S.D.	LRuc	ABLF	AQLF	UL
Normal	95% VaR	-3.9342	1.6231	0.1041	0.0516	0.1878	-0.0572
	99% VaR	-5.6043	2.3048	0.8384	0.0122	0.0482	-0.0130
	99.5% VaR	-6.2157	2.5544	0.4169	0.0061	0.0293	-0.0082
GED	95% VaR	-3.9322	1.6056	0.4124	0.0533	0.1885	-0.0567
	99% VaR	-5.8702	2.3483	0.5354	0.0083	0.0368	-0.0101
	99.5% VaR	-6.6131	2.6327	0.4838	0.0038	0.0199	-0.0061
SGT	95% VaR	-3.9221	1.5924	0.0130	0.0494	**0.1855**	**-0.0565**
	99% VaR	-6.0281	2.3227	2.1626	0.0067	**0.0283**	**-0.0092**
	99.5% VaR	-6.9432	2.6341	0.0957	0.0044	**0.0119**	**-0.0041**

Notes: * represents significance under 1% level. LRuc is the Log-likelihood test for correct unconditional coverage. ABLF is the average binary loss function. AQLF is the average quadratic loss function. UL denotes the unexpected loss, which refers to the average dollar loss caused by the failures of VaR model.

Table 4. Out-of-sample performance of alternative distributions for gold

Part A. Spot

		Mean	S.D.	LRuc	ABLF	AQLF	UL
Normal	95% VaR	-2.5769	1.4962	3.9672*	0.0605	0.3658	-0.0733
	99% VaR	-3.6759	2.1151	28.8773**	0.0250	0.1984	-0.0322
	99.5% VaR	-4.0783	2.3416	38.0757**	0.0183	0.1651	-0.0244
GED	95% VaR	-2.4878	1.3122	5.2706**	0.0622	0.3950	-0.0792
	99% VaR	-4.2121	2.1530	8.9335**	0.0177	0.1604	-0.0231
	99.5% VaR	-4.9446	2.5072	3.3448**	0.0083	0.1187	-0.0146
SGT	95% VaR	-2.4837	1.1153	2.4477	0.0587	**0.4427**	**-0.0804**
	99% VaR	-4.5451	1.9566	0.9358	0.0125	**0.1669**	**-0.0211**
	99.5% VaR	-5.6366	2.4104	0.5503	0.0037	**0.1078**	**-0.0130**

Part B. Futures

		Mean	S.D.	LRuc	ABLF	AQLF	UL
Normal	95% VaR	-1.8894	0.6966	2.1878	0.0577	0.1329	-0.0459
	99% VaR	-2.6959	0.9880	18.5572**	0.0216	0.0500	-0.0182
	99.5% VaR	-2.9911	1.0947	28.0883**	0.0161	0.0355	-0.0133
GED	95% VaR	-1.8807	0.7054	2.1878	0.0577	0.1333	-0.0461
	99% VaR	-2.9817	1.1008	5.7296*	0.0161	0.0368	-0.0137
	99.5% VaR	-3.4252	1.2600	8.4500**	0.0105	0.0223	-0.0085
SGT	95% VaR	-1.9780	0.8220	0.1156	0.0518	**0.1256**	**-0.0438**
	99% VaR	-3.3187	1.4320	1.8863	0.0135	**0.0311**	**-0.0115**
	99.5% VaR	-3.9548	1.7703	2.5151	0.0080	**0.0171**	**-0.0061**

Notes: ** and * represent significance under 1% and 5% level. LRuc is the Log-likelihood test for correct unconditional coverage. ABLF is the average binary loss function. AQLF is the average quadratic loss function. UL denotes the unexpected loss, which refers to the average dollar loss caused by the failures of VaR model.

Table 5. Descriptive statistics of time-varying scaling parameters

			Mean	S.D.	Min.	Max.
Part A. Crude oil						
Spot:	GED ~ kurtosis:	κ	1.315	0.198	0.916	1.779
	SGT ~ kurtosis:	κ	2.347	0.335	1.566	4.371
		n	9.993	14.033	3.732	183.453
	skew:	λ	-0.049	0.044	-0.151	0.101
Futures:	GED ~ kurtosis:	κ	1.221	0.179	0.913	1.585
	SGT ~ kurtosis:	κ	2.204	0.388	1.444	3.742
		n	87.476	121.474	3.770	363.980
	skew:	λ	-0.054	0.055	-0.160	0.132
Part B. Gold						
Spot:	GED ~ kurtosis:	κ	1.898	0.147	1.567	2.289
	SGT ~ kurtosis:	κ	1.934	0.527	1.062	5.294
		n	4.994	3.405	2.048	70.813
	skew:	λ	-0.094	0.081	-0.285	0.080
Futures:	GED ~ kurtosis:	κ	1.531	0.239	1.168	2.244
	SGT ~ kurtosis:	κ	2.001	0.574	1.065	5.479
		n	31.372	68.244	2.001	290.703
	skew:	λ	-0.127	0.126	-2.183	0.165

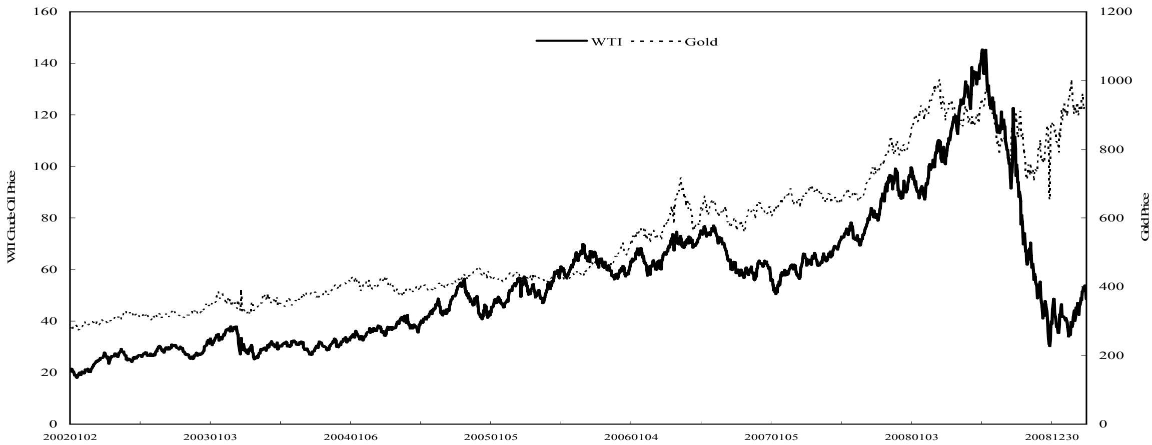
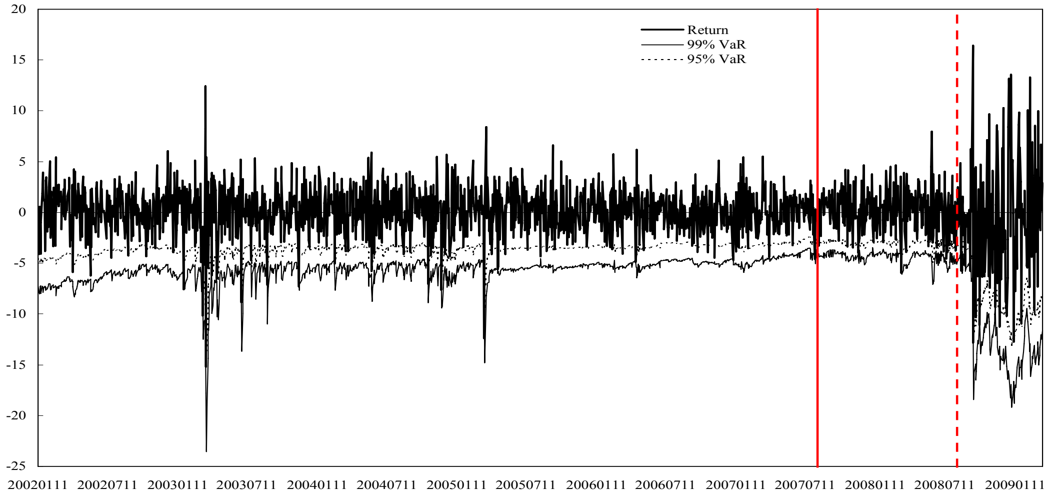
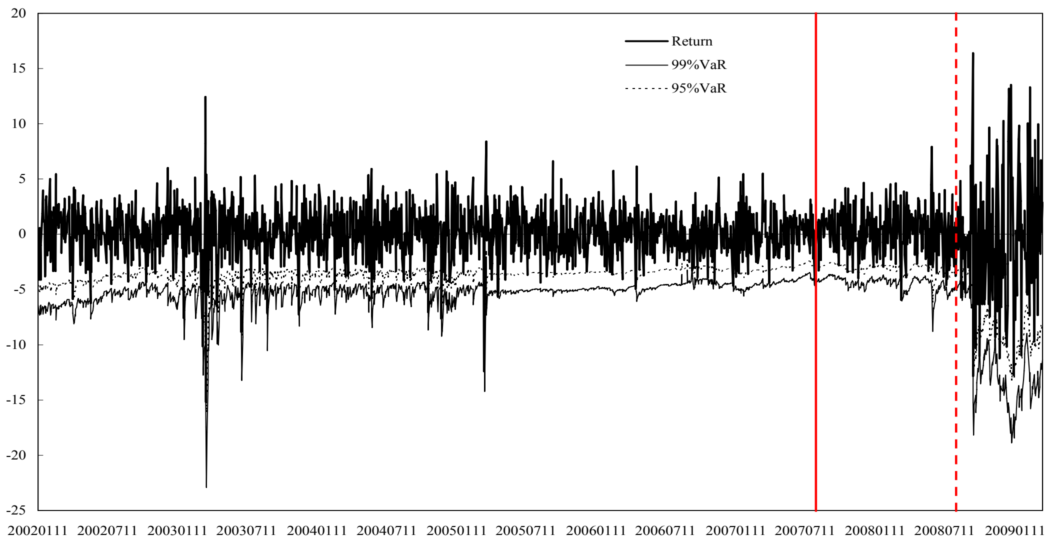


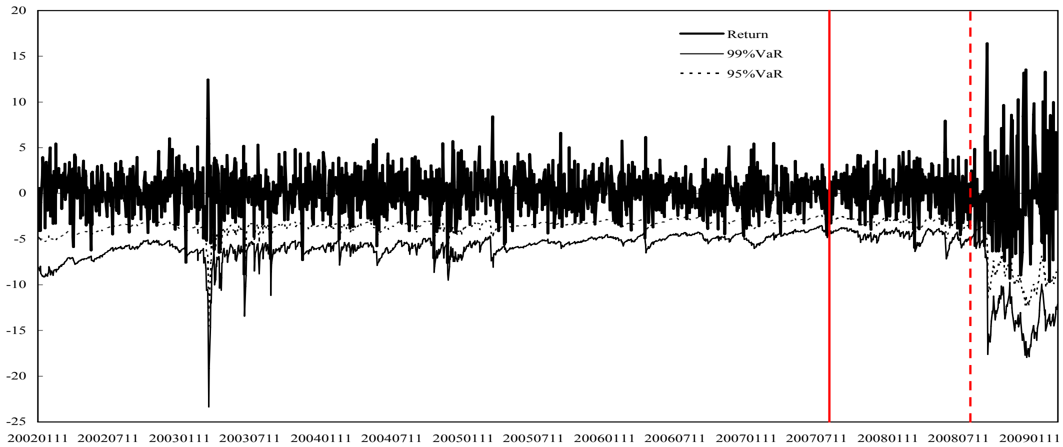
Figure 1. The time series plot of crude oil and gold



Part A. GED distributions

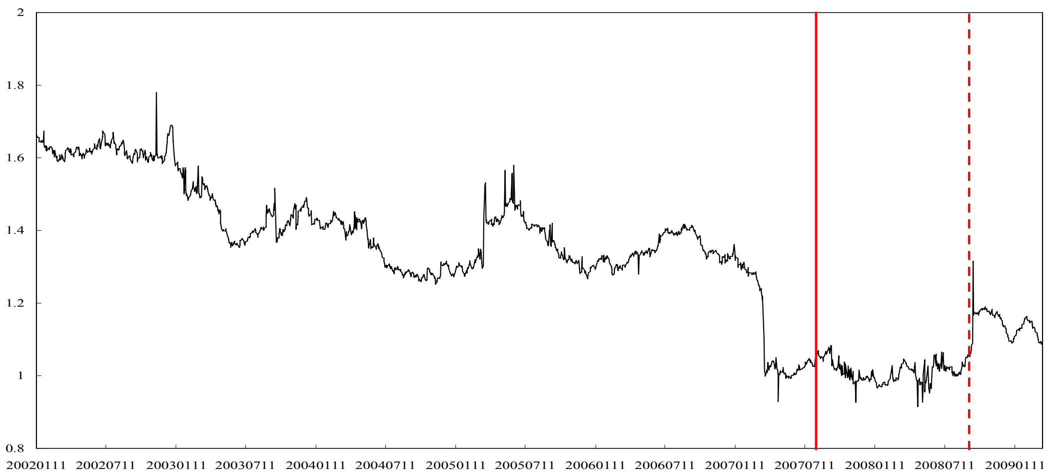


Part B. Normal distributions

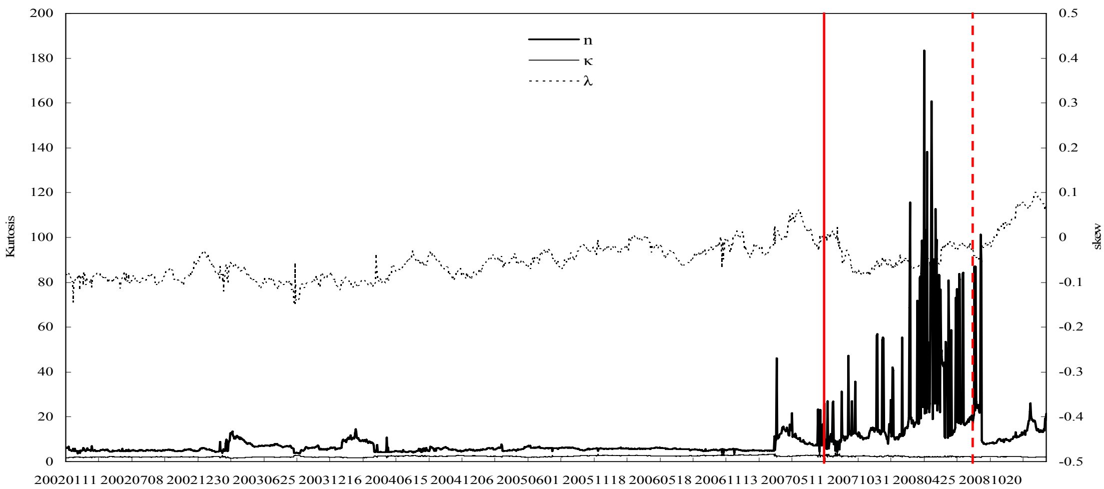


Part C. SGT distribution

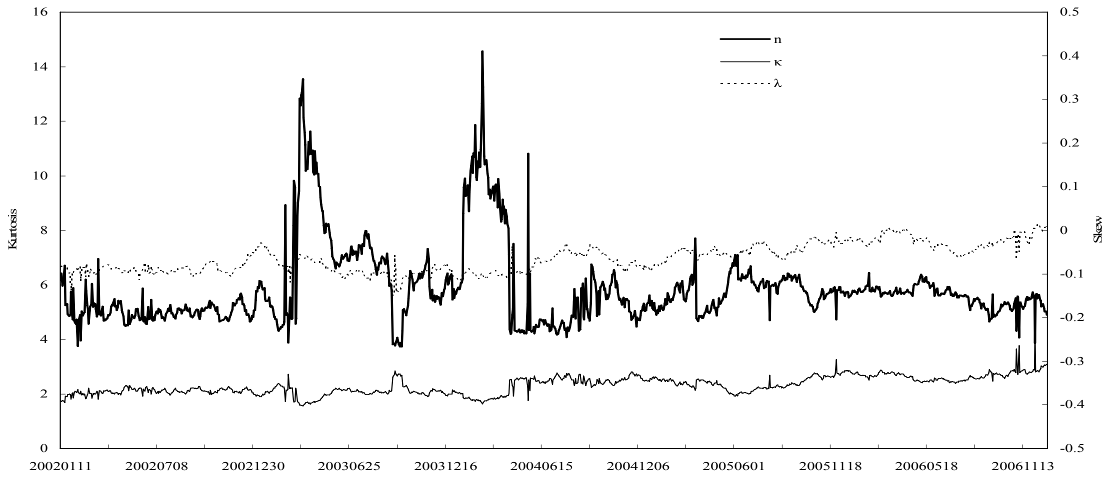
Figure 2. Forecasted VaR with different distributions in crude oil spot



Part A. Kurtosis parameter in the GED distribution

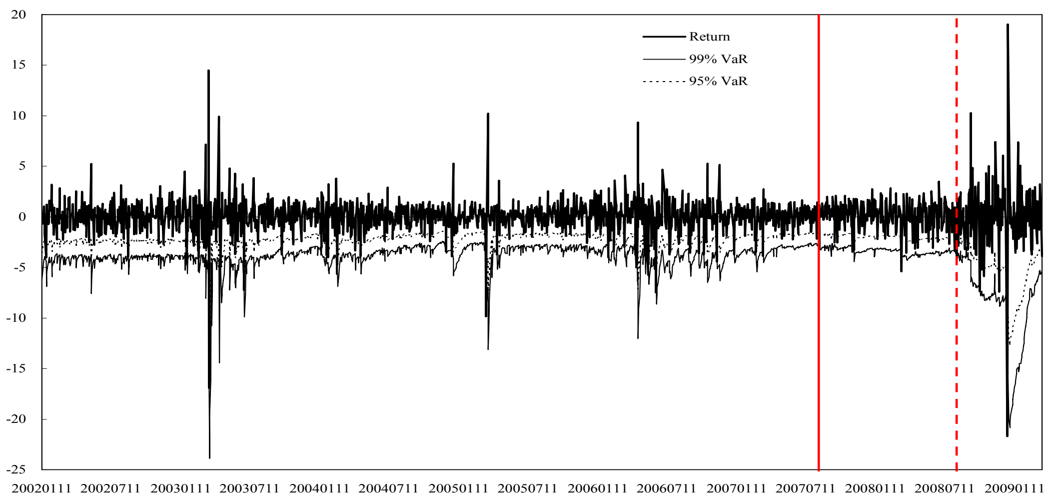


Part B. Skewness and kurtosis parameters in the SGT distribution

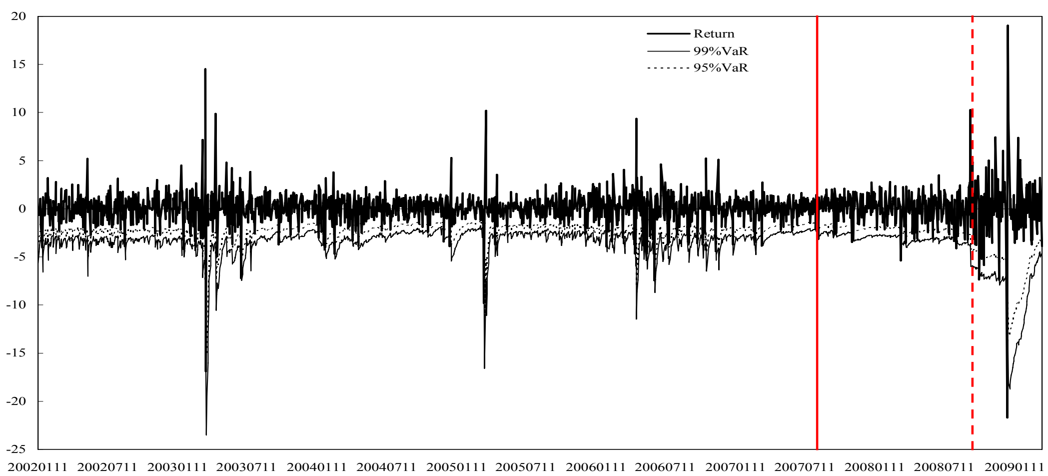


Part C. Skewness and kurtosis parameters in the SGT distribution exclude the period of global financial crisis

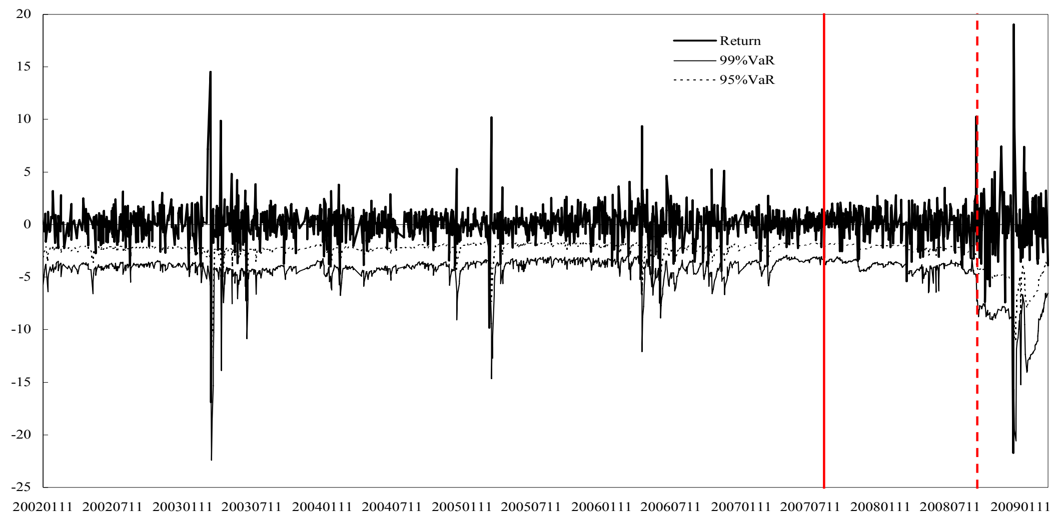
Figure 3. The time-varying scaling parameters



Part A. GED distribution

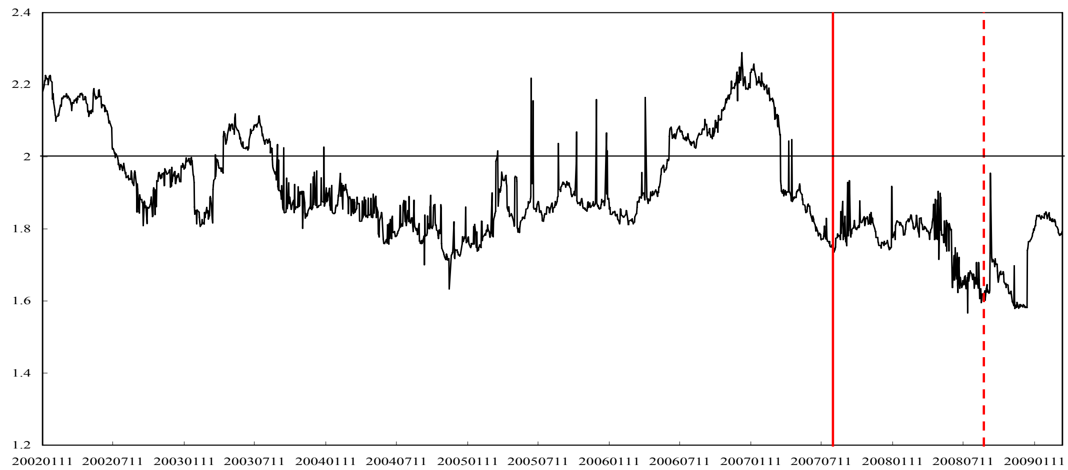


Part B. Normal distribution

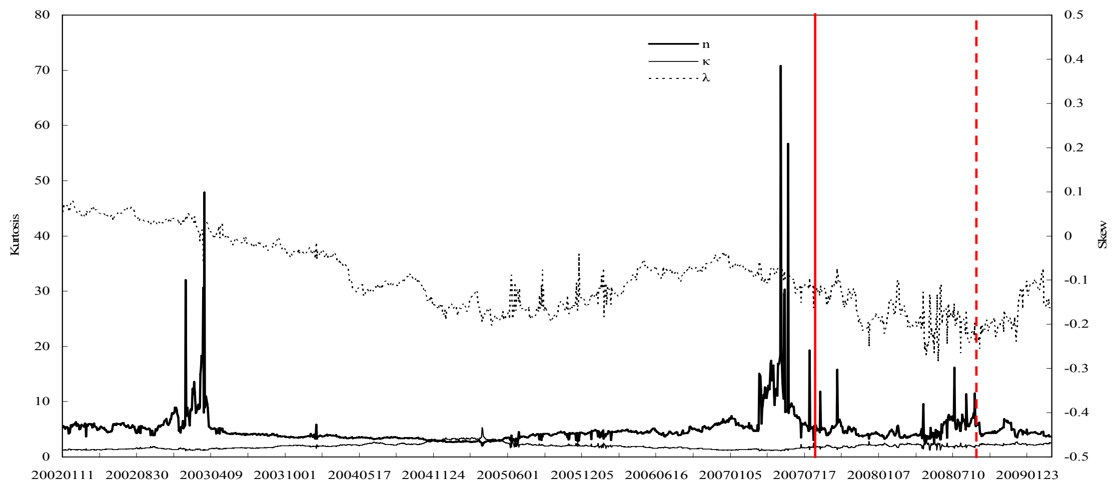


Part C. SGT distribution

Figure 4. Forecasted VaR with different distributions in gold spot



Part A. Kurtosis parameters in GED distribution



Part B. Skewness and kurtosis parameters in the SGT distribution

Figure 5. The time-varying scaling parameters

出席國際學術會議心得報告

計畫編號	NSC 97-2410-H-343-005-
計畫名稱	商品波動與經濟活動
出國人員姓名 服務機關及職稱	鄭婉秀 淡江大學財務金融學系　助理教授
會議時間地點	2009.6.21-24　Hong Kong
會議名稱	2009 International Symposium on Forecasting
發表論文題目	Distributed and Forecasted in Commodity Volatility

一、參加會議經過

「2009 International Symposium on Forecasting」是由International Institute of Forecasting (IIF)舉辦的第29屆國際研討會，舉辦地點在香港尖沙咀喜來登飯店，自6月21至24日，為期4天，約有近200篇論文被接受且發表。22-24日每天早上皆有一場Keynote Speech及Feature speech，三場Keynote Speech分別為：

(1) Time Varying Parameter Structural Time Series Models: An Application to Tourism Demand by Professor Stephen F. Witt, Hong Kong Polytechnic University, Hong Kong SAR, China

(2) Extreme Forecasting by Professor Rob Hyndman, Monash University, Australia

(3) Evidence-based Methods to Forecast Elections: The PollyVote Project by Professor Scott Armstrong, Wharton School of Business, University of Pennsylvania, USA

其餘時間皆為論文發表，每個場次約1小時，安排三篇文章發表，報告15分鐘，提問5分鐘。本人論文被安排在最後一天主題為Finance的場次中，每篇paper都有人提問，Chairman除控制時間外，也一定會提問題，會後學者們也各自針對有興趣的主題交換意見及名片。會議的網站資料十分完善，相關資訊都可以自網站查詢得到，十分便利。

二、與會心得

出席本次會議對我而言是相當難得的經驗，也是相當大的挑戰。本人求學過程都在台灣國內，從未出國歷練過，這是本人第一次參與國際會議，在任何方面均是非常大的震撼教育，語言方面是最大的挑戰，相較於香港的學生及學者，其面對英文得以侃侃而談的程度，本人時應該多多加強。由於平時少有英文環境，在面對國外學者提問時，總有無法把所學完整表達之憾。因此，本人自我期許除平時多加強以英文思考外，也要多參加國際會議，除了瞭解目前的新研究議題，提升研究能量外，也可以克服對英文環境的恐懼感，是一個非常好的訓練。

相較於台灣，香港真是一個非常國際化的都市，一進到機場，感覺與台灣明顯不同，香港學校對學生的訓練(尤其是博士班)也相對的非常有國際觀，其論文表達能力對我而言都是榜樣，是一個值得台灣學生學習的方向。

Skewness and Leptokurtosis in VaR Estimation:

An Analysis Focusing on Commodity Asset Returns

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Abstract

This paper use the flexible skewed generalized t distribution (SGT) to provide an accurate characterization of the non-normal of the commodity return distributions, and analyze the time-varying scaling parameters, including those in the crude oil and gold markets. We also estimate the VaR on the basis of the GARCH-SGT model, the out-of-sample forecasting periods covers a long period, including the most unsteady period of the global financial crisis. The empirical results show that the forecasted VaR with the SGT distribution provides the most accurate out-of-sample forecasts either in the crude oil or gold markets. In the crude oil market, though all the distributions provide the correct coverage rate, the forecasted oil spot VaR with the normal distribution is appropriate only for the low confidence level, and the accuracy and performance deteriorate and lose accuracy for the higher confidence levels. In comparison, the SGT distribution provides the most accurate out-of-sample forecasts within the strict VaR confidence levels. With regard to the gold markets, the most appropriate distribution for the forecasted VaR is the SGT distribution, and the failure rates in the normal distribution and GED for the VaR are statistically higher than the specific probability of the model. Comparatively, the scaling parameters in the SGT distribution can capture the volatilities of oil and gold effectively and they show that the unexpected losses are smaller in the SGT distribution. Finally, the estimated VaR within the SGT model is significantly superior to the other distributions in the crude oil and gold markets.

Keywords: Skewed generalized t distribution; Commodity volatility; Value-at-Risk

INTRODUCTION

Most time series are characterized by leptokurtosis and skewness, not only in financial assets (Bollerslev, 1987; Engle and Gonzales-Rivera, 1991; Ait-Sahalia and Lo, 1998; Theodossiou and Trigeorgis, 2003; Bali and Theodossiou, 2007) but also in energy assets (Solt and Swanson, 1981; Taylor, 1998; Giot and Laurent, 2003; Chan et al., 2007; Fan, 2008). Moreover, empirical evidence has shown that the conditional normal time series models are inadequate for estimating the conditional return distribution. However, relatively little work has been carried out on modeling and estimating volatilities in oil and gold assets by using non-normal distributions, for example, in oil markets. Giot and Laurent (2003), Chan et al. (2007), Fan et al. (2008), and Hung et al. (2008) comprise the limited body of work that calculate the Value-at-Risk (VaR) of commodity assets using non-normal distributions; a majority of the studies that measure the volatility of oil returns do so with normal distributions (Cabedo and Moya, 2003; Busch, 2005; Sadorsky, 2006; Sadeghu and Shavvalpour, 2006). Fan et al. (2008) pointed out that it is important to be acquainted with the characteristics of oil market risks. However, the available quantitative literatures, for example, in the gold market, are very limited. Casassus and Collin-Dufresne (2005) recently evaluated the VaR for gold, using a three factor model. This is unfortunate given the importance of oil and gold to the global economy. For participating in oil and gold markets, it is also crucial to describe the asset prices; however, no appropriate method is available for this purpose. Volatility is the principal factor for developing the economic and financial models of pricing and hedging, and estimations made under the correct specifications of the conditional distribution are more efficient. Therefore, this paper utilizes the most flexible distribution to describe the oil and gold volatilities that are characterized by leptokurtosis and skewness.

To measure market risk, the application of the VaR methodology offers comprehensive and recapitulative advantages. In practice, a key risk measure based on the VaR concept is the conditional VaR, which is the worst possible loss at a given confidence level due to adverse market movements over the next reporting period, conditional on the current portfolio volatility and market information. Mathematically, VaR is defined as a quantile of a probability distribution, used to model an underlying portfolio value or its return. It is commonly used in symmetric and normal distributions for asset returns. Portfolio VaR is often calculated on the basis of the variance-covariance approach, and returns follow the normal distribution. The most used models are the classical autoregressive conditional heteroscedasticity (ARCH)/generalized ARCH (GARCH) models, with attributes such as volatility clustering and the long-range dependence structure that exist in financial assets; moreover, these models are based on conditional Gaussian innovations (see Engle, 1982; Bollerslev, 1986). However, empirical evidence has demonstrated that the conditional normal time series models are inadequate for estimating the tail quantiles of the conditional return distributions. Substantial empirical evidence shows that the distribution of financial returns is typically skewed, peaked around the mean (leptokurtic) and characterized by fat tails. Bollerslev, Engle, and Nelson (1994) proposed that the leptokurtosis is reduced, but not eliminated, when returns are standardized using time-varying estimates for the means and variances. This prompts the gradual adoption of models with heavy-tailed innovations in risk modeling. Many extensions of the classical GARCH models with heavy-tailed innovations have been proposed.

Student's t, generalized error distribution (GED), and a mixture of two normal distributions are frequently used for describing the non-normal characteristics in the VaR literature. With regard to the commodity markets, Giot and Laurent (2003) compared the performance of the RiskMetrics, skewed Student asymmetric power GARCH (APGARCH), and skewed Student ARCH models for several commodities. They found that the skewed Student ARCH model delivered excellent results and was relatively easy to use. Chan et al. (2007) considered a GARCH model with heavy-tailed innovations and characterized the limiting distribution of an estimator of the conditional VaR, which corresponds to the external quantile of the conditional distribution of the GARCH process. Fan et al. (2008) estimated the VaR of the returns in West Texas Intermediate (WTI) and Brent crude oil spot markets using a GED-GARCH model. They found this approach to be more realistic and comprehensive than the commonly used standard normal distribution-based VaR model, and also more effective than the well-recognized historical simulation with autoregressive moving average (ARMA) forecasts. Hung et al. (2008) investigated the fat-tailed innovation process on the VaR estimates, and the empirical results showed that the GARCH-HT model is quite accurate and efficient in estimating the VaR for energy commodities.

However, because such distributions partially deal with the issues of leptokurtosis and skewness, they cannot fully correct the measurement bias in risk problems (Bali and Theodossiou, 2007). The skew generalized t (SGT) distribution, introduced by Theodossiou (1998), is a skewed extension of the generalized-t distribution, originally proposed by McDonald and Newey (1988). The SGT is a distribution that allows for a very diverse level of skewness and kurtosis, and it has been used to model the unconditional distribution of daily returns for a variety of financial assets (Theodossiou, 1998; Harris and Kucukozmen, 2001). Furthermore, the SGT nests several well-known distributions such as the generalized t (GT) of MacDonald and Newey (1988); the skewed t (ST) of Hansen (1994); the skewed generalized error distribution (SGED) of Theodossiou (2001); and the normal, Laplace, uniform, GED, and student t distributions. Harris et al. (2004) further found that a conditional SGT distribution offers a substantial improvement in the fit of the GARCH model for stock index assets. Bali and Theodossiou (2007) proposed a conditional technique for estimating the VaR and expected shortfall measures on the basis of the SGT distribution in the S&P 500 index returns. They found that GARCH-type models with the SGT distribution are much superior to the conditional normal distribution for all GARCH specifications and all probability levels. Bali et al. (2008) also used the SGT distribution with time-varying parameters to provide an accurate characterization of the tail of the standardized equity return distributions. To fill in the gap in the inadequate research in which the SGT distribution in non-normal commodity returns has been employed, we use the GARCH-SGT model to model the commodity volatilities. The analytical and empirical results in this paper could provide better approximations of reality.

The remainder of this paper is organized as follows. Section 2 describes the motivation behind focusing on the crude oil and gold markets. Section 3 presents the methodologies of the GARCH-SGT models and the measurement of the VaR. Section 4 compares the out-of-sample empirical results of the SGT, and the normal distribution and GED. Section 5 concludes the paper.

IMPORTANCE OF CRUDE OIL AND GOLD

Oil is one of the most important commodities, and almost everything tangible that we physically move burns oil in the process. One of the characteristics of the oil market prices is volatility, which is both high and variable over time. In general, oil prices have become more volatile since 1986 (Plourde and Watkins, 1998; Lynch, 2002; Regnier, 2007), and this volatility has a significant impact on the global economy (Lee et al., 1995; Ferderer, 1996; Sadorsky, 1999, 2006). US oil prices have been heavily regulated through production or price control measures throughout much of the twentieth century. With the exception of the occasional jump in late 1990, crude oil prices have risen progressively since the later part of 2001. In January 2008, oil prices unprecedentedly surpassed $100 a barrel, the first of many price milestones to be passed in the course of the year. In July 2008, oil prices peaked at $147.30 a barrel. In the second half of 2008, the prices of most commodities fell dramatically in anticipation of diminished demand owing to the recent global recession. In fact, these high prices resulted in a dramatic drop in demand and prices fell below $35 a barrel at the end of 2008. It is believed that high prices will cause genuine economic damage, resulting in the threat of stagflation and a reversal of globalization. In July 2009, the president of the Organization of Petroleum Exporting Countries (OPEC), Jose Maria Botelho de Vasconcelos, remarked that a crude oil price of $68–$71 a barrel was optimal for a stable industry. The oil market was very fragile, and crude prices were susceptible to huge fluctuations caused by minor events. Factors such as high demand, low supply, strategies adopted by OPEC, environmental regulations, hedge fund actions, and violence in the Middle East have all stimulated prices. A traditional demand-based framework was unable to explain the marked deterioration in the commodity and oil prices (Chaudhuri, 2001). Jalali-Naini and Manesh (2006) also pointed out that high volatility is a very promising characteristic for testing volatility models.

Of all the precious metals, gold is the most popular as an investment. Investors generally buy gold as a hedge or safeguard against any economic, political, social, or currency-based crises. History has shown that in adverse periods, investors tried to preserve their assets by investing in precious metals, most notably gold and silver. Since April 2001, the gold price has more than tripled in value against the US dollar, prompting speculation that this long secular bear market (or the Great Commodities

Depression) has ended and a bull market has reemerged. In March 2008, the gold price increased above $1,000. A number of studies have reported on the relationship between gold and macroeconomic variables (Sherman, 1983; Baker and Van-Tassel, 1985; Kaufmann and Winters, 1989; Sjaastad and Scacciavillani, 1996; Taylor, 1998; Christie-David et al., 2000; Cai et al., 2001; Tully and Lucey, 2006). These studies confirmed that macroeconomic variables such as the exchange rate of dollar, stock index, interest rate, consumer price index (CPI), and unemployment rate influence gold returns. In contrast, Lawrence (2003) argued that no significant correlations exist between gold returns and changes in certain macroeconomic variables.

To address the ambiguous empirical results in measuring the VaR within oil and gold markets, this paper provides a comprehensive analysis using the flexible SGT distribution for modeling the volatilities. This paper extends the existing research in oil and gold markets in four important ways. First, we calculate the VaR on the basis of the SGT—a distribution that allows for a very diverse level of skewness and kurtosis—for modeling the distribution of commodity returns. The normal distribution and GED are the comparable models used to assess the robustness of the SGT distribution. Second, considering the behavior of highly volatile oil and gold assets, we employ the GARCH models for estimating the time-varying conditional variance of returns. Third, we analyze the time-varying scaling parameters of crude oil and gold assets. It will be easy to observe why traditional distributions are not appropriate for estimating volatilities and forecasting the VaR. Fourth, this paper investigates the volatility in the prices—both spot and futures—of oil and gold assets. This paper also analyzes the performance of out-of-sample forecasting for a long period, covering both stable and high-fluctuation periods, including the period of the current global financial crisis. The VaR in the SGT distribution is significantly superior to other distributions.

METHODOLOGY

GARCH(1,1) Model with Skewed Generalized T Distribution (GARCH-SGT)

This paper investigates GARCH(1,1) model in computing the conditional means and conditional variances for conditional VaR analysis. The GARCH(1,1) model proposed by Bollerslev (1986) is as follows:

$$r_t = \mu_t + \varepsilon_t, \ \ \varepsilon_t \sim (0, h_t) \tag{1}$$

$$h_t = \beta_0 + \beta_1 h_{t-1} z_{t-1}^2 + \beta_2 h_{t-1} \tag{2}$$

where $\beta_0 > 0$, $\beta_1 > 0$, $\beta_2 > 0$ and $\beta_1 + \beta_2 \le 1$. In the equations, μ_t and h_t are the conditional mean and conditional standard variance of returns r_t based on the information set Ω_{t-1} up to time t-1. The standardized error term is $z_t = \varepsilon_t \big/ \sqrt{h_t}$.

Considering the non-normal characteristics of energy assets, the conventional GARCH model with normal distribution is fail to capture the behavior of high-volatility of oil and golf assets. SGT distribution, advanced by Theodossiou (1998), is displaced for well-describing the distribution of assets returns exhibiting skewness and leptokurtosis. The probability density function for the SGT distribution can be represented as follows:

$$f(z_t | n, \kappa, \lambda) = C\left(1 + \frac{|z_t + \delta|^{\kappa}}{((n+1)|\kappa)(1 + \text{sign}(z_t + \delta)\lambda)^{\kappa}\theta^{\kappa}}\right)^{-\frac{n+1}{\kappa}} \tag{3}$$

where $C = 0.5\kappa\left(\frac{n+1}{\kappa}\right)^{-\frac{1}{\kappa}} B\left(\frac{n}{k}, \frac{1}{k}\right)^{-1} \theta^{-1}$, $\theta = \frac{1}{\sqrt{g - \rho^2}}$, $\delta = \rho\theta$,

$$\rho = 2\lambda B\left(\frac{n}{\kappa}, \frac{1}{\kappa}\right)^{-1} \left(\frac{n+1}{\kappa}\right)^{\frac{1}{\kappa}} B\left(\frac{n-1}{\kappa}, \frac{2}{\kappa}\right)$$

$$g = (1 + 3\lambda^2) B\left(\frac{n}{\kappa}, \frac{1}{\kappa}\right)^{-1} \left(\frac{n+1}{\kappa}\right)^{\frac{2}{\kappa}} B\left(\frac{n-2}{\kappa}, \frac{3}{\kappa}\right)$$

where λ is a skewness parameter, "sign" is the sign function, B(·) is the beta function, and δ is the Pearson's skewness and mode of $f(z_t)$. The scaling parameters n, κ and λ obey the following constraints: $n > 2$, $\kappa > 0$ and $-1 < \lambda < 1$. The skew parameter λ controls the rate of descent of the density around the mode of z. In the case of positive skewness ($\lambda > 0$), the density function is skewed to the right. In contrary, the density function is skew to the left with the negative skewness ($\lambda < 0$). The parameter n and κ control the tail and height of the density. Smaller values of κ and n result in larger values for the kurtosis (i.e. more leptokurtosis p.d.f.s) and vice versa. The parameter κ (>0) determines the (fat) tail and height or shape (degree of leptokurtosis) of the distributions (eq., normal for $\kappa = 2$ and Laplace for $\kappa = 1$; thinner tail than normal for $\kappa > 2$ vs. thicker tail than normal for $\kappa < 2$). The parameter n has the degree of freedom interpretation in the case $\lambda = 0$ and $\kappa = 2$. Moreover, larger positive values of λ result in larger positive values for both skewness and kurtosis (Theodossiou, 1998).

The SGT distribution nests several well-known distributions (see Table 1). Specifically, it gives for $\lambda = 0$, McDonald's and Newey's(1988) GT distribution; for $\kappa = 2$, Hansen(1994)'s skewed student's *t* distribution; for $\lambda = 0$ and $\kappa = 2$, the student's *t* distribution; for $\lambda = 0$ and $n = \infty$, the Subbotin(1923)'s power exponential distribution; for $\lambda = 0$, $\kappa = 1$ and $n = \infty$, the Laplace distribution; for $\lambda = 0$, $\kappa = 2$ and $n = 1$, the Cauchy distribution; for $\lambda = 0$, $\kappa = 2$, and $n = \infty$, the normal distribution; and for $\lambda = 0$, $\kappa = \infty$, and $n = \infty$, the uniform distribution. Furthermore, the conditional version of SGT for $\kappa = 2$ nests the conditional skewed t distribution of Jondeau and Rockkinger (2003).

The log-likelihood function of the GARCH-SGT model can be written as:

$$\underset{SGT}{\mathrm{LogL}} = \sum_{t=1}^{T} \ln f(z_t | n, \kappa, \lambda) \tag{4}$$

Measurement and Performance in VaR

Definition and estimation

A VaR model measures market risk for a portfolio of financial assets and measures the potential loss that a portfolio could lose over a given period of time. The manager may be interested in making a statement of the following form: "We are p percent certain that we will lose more than υ dollars in the next N days." The variable υ_t is the VaR of the portfolio. Mathematically, the function can be expressed as:

$$p = \int_{-\infty}^{\upsilon_t} f_t(r)dr, \tag{5}$$

where $f_t(r)$ represents the probability density function of return r_t, the change in the value of a portfolio over a certain horizon N days. The one-day-ahead VaR based on the GARCH-SGT can be calculated as:

$$VaR_{t+1}^{SGT} = f_\alpha(z_t; n, \kappa, \lambda) \cdot \sqrt{h_t} - E(r) \tag{6}$$

where $f_\alpha(z_t; n, \kappa, \lambda)$ denotes the left quantile at α for SGT distribution[1] with scaling parameters n, κ and λ. The h_t is the conditional variance of the GARCH model.

Test of correct conditional coverage

A "failure" is defined as an outcome $r_t < \upsilon_t$. Intuitively, a "good" VaR estimators υ_t would be such that $\Pr(r_t < \hat{\upsilon}_t)$ is close to p. The indicator variable is set as followed,

$$I_t = \begin{cases} 1, & \text{if} \quad r_t < \upsilon_t \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

[1] The quantiles of the SGT distribution with various combinations of shape parameters are calculated with numerical integration or bootstrapping technique.

The stochastic process $\{I_t\}$ is called the failure process. The VaR forecasts are said to be efficient of they display correct conditional coverage, that is, $E(I_{t|t-1}) = p \; \forall \; t$. Kupeic (1995) develops a test for correct unconditional coverage in the likelihood ratio (LR) framework. The likelihood ratio statistics is as follows:

$$LR_{uc} = -2\log\left[\frac{p^{n_1}(1-p)^{n_0}}{\hat{\pi}^{n_1}(1-\hat{\pi}^{n_0}}\right] \sim \chi^2_{(1)} \tag{8}$$

where p is the tolerance level where VaR measures are estimated, n_1 (n_0) is the number of 1 (0) in the indicator series, and $\hat{\pi} = n_1/(n_1 + n_0)$, the MLE of p. The null hypothesis of the failure probability p is tested against the alternative hypothesis that the failure probability is different from p.

Evaluation using regulatory loss function

The loss function evaluation method proposed based on assigning to VaR estimates a numerical score that reflects specific regulatory concerns. It provides a measure of relative performance that can be used to monitor the performance of VaR estimates. Two regulatory loss functions proposed by Lopez (1998) are described below.

(1) Binary loss function

If the predicted VaR is not able to cover the realized loss, this is termed a 'violation'. A binary loss function is merely the reflection of the LR test of unconditional coverage test and gives a penalty of one to each exception of the VaR. Namely,

$$L_{t+1} = \begin{cases} 1, & \text{if} \quad r_{t+1} < \upsilon_{t+1} \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

If a VaR model truly provides the level of coverage defined by its confidence level, then the average binary loss function over the full sample will equal p for the (1-p) th percentile VaR

(2) Quadratic loss function

The quadratic loss function of Lopez (1998) penalizes violations differently from the binary loss function, and pays attention to the magnitude of the violation. That is,

$$L_{t+1} = \begin{cases} 1+(r_{t+1}-\upsilon_{t+1})^2 \text{ , if } r_{t+1} < \upsilon_{t+1} \\ 0 \qquad\qquad\qquad\quad \text{, if } r_{t+1} \geq \upsilon_{t+1} \end{cases} \tag{10}$$

The quadratic term ensures that large violations are penalized more than the small violations which provide a more powerful measure of model accuracy than the binary loss function.

EMPIRICAL RESULTS

Out-of-sample forecasting performance: Crude oil markets

To assess the forecasting performance with alternative distributions, we first make estimates on the basis of daily returns for two years, after which the estimation period is continuously rolled forward by adding the most recent day and excluding the oldest. Following this process, the out-of-sample VaRs are calculated for the next 1,800 days (from January 2002 to March 2009); the results are illustrated in Figure 2. The forecasting performance can be analyzed in terms of the integrity of the results for the long forecasting period, which includes both stable and high volatility periods, especially through the global financial crisis period from 2007. Tables 3 and 4 list the out-of-sample forecasting results for crude oil and gold in this paper; Panels A and B show the spot and futures prices, respectively.

We first discuss the results of the crude oil spot market. All the statistics are not significant in the correct unconditional coverage test (LRuc), thus indicating that the estimated failure probability is statistically consistent with the specified probability of the model. We then compare the unexpected loss (UL) and the average quadratic loss function (AQLF[2]). For the low confidence level (95% VaR), the normal distribution yields the highest VaR estimates and the lowest failure rates in the AQLF and UL. In comparison, the failure rates in the GED and SGT distribution are higher for the low confidence level of 95% VaR. Although these distributions provide a correct coverage rate, the SGT distribution and GED have lower accuracy than normal distribution. However, the results are completely different for the high confidence levels (99% and 99.5%). The most correct VaR estimates are with the SGT distribution, whereas the accuracy and performance with the GED and normal distribution deteriorate and lose accuracy. A comparison of the AQLF values reveals that the lowest value is 0.0597 and 0.0362 for 99% and 99.5%, respectively, with the SGT distribution; 0.0699 and 0.0445, respectively, with the GED; and 0.0785 and 0.0542, respectively, with the normal distribution. It is obvious that the SGT distribution provides the most accurate out-of-sample forecasts within the strict VaR confidence levels. With regard to the crude oil futures prices, the SGT distribution shows the best performance in any confidence level. Part B in Table 3 shows that although all the estimated failure rates are statistically consistent with the specified probability of the model, the AQLF and UL are the lowest with the SGT distribution. The best (or lowest) AQLF for 95%, 99%, and 99.5% VaR is 0.0185, 0.0283, and 0.0119, respectively, with the SGT distribution, and the inferior AQLF is with the GED; further, the worst (or highest) AQLF is 0.1878,

2 AQLF is the final standard when we select the best model, because large violations are penalized more than the small violations, and it provides a more powerful measure of model accuracy than other standards.

0.0482, and 0.0293, respectively, with the normal distribution. In sum, the skewness and leptokurtosis result in the improper VaR estimates with the normal distribution.

Next, we illustrate the time-varying scaling parameters in Figure 3 for analyzing the superiority of the SGT distribution in the crude oil spot prices[3]. Two lines are drawn: the solid line indicates the values as on August 9, 2007, and the dotted line indicates the values as in September 2008. The former date indicates the beginning of the global financial crisis, which resulted in a liquidity crisis that prompted a substantial injection of capital into the financial markets by the United States Federal Reserve, Bank of England, and the European Central Bank. In the latter date, September 2008, the crisis deepened, as stock markets worldwide crashed and entered a period of high volatility, and a large number of banks, mortgage lenders, and insurance companies failed in the subsequent weeks. In the part in the figure indicating the GED, we can see that the fat-tail parameter (κ) is below 2 in the forecasting period, indicating that the fat-tail exists in the crude oil spot prices. However, the fluctuation of the parameter κ is not large, except in the global financial crisis period when it is comparatively low. In comparison, an observation of the scaling parameters of the SGT distribution in Part B of Figure 3 shows that the skewness parameter λ is smooth around 0, thus indicating that the skewness is not very important. However, two kurtosis parameters (κ and n) perform differently in the forecasting period. The first parameter, κ, is very smooth and the average value is close to that of the normal distribution (i.e., 2), indicating no peakness for the empirical distribution. The second parameter, n, on the other hand, is very volatile, especially in the beginning of the global financial crisis. By definition, the smaller values of κ and n result in larger values for the kurtosis, and vice versa, and the SGT distribution is close to the normal distribution while $\lambda = 0$, $\kappa = 2$, and $n = \infty$. We can therefore say that the normal

[3] The results are shown in terms of crude oil futures prices.

distribution is appropriate only for the starting year of the global financial crisis and the fat-tail distribution is more appropriate for other periods. To eliminate the extremely high value for parameter n, we redraw the scaling parameters in Part C in Figure 3. It can be clearly seen that the fluctuation in parameter n and the estimated values of the parameter were not very large except in mid-2003 and early 2004, indicating significant fat tails for the empirical distribution of standardized returns. This is why the forecasting performance with the normal distribution was not better than that with the alternative distributions.

Out-of-sample forecasting performance: Gold markets

Next, we analyze the gold spot market, for all the confidence levels. The estimated failure rates with the SGT distribution are the only ones that pass the coverage rate test, LRuc. However, the estimated failure rates with either the normal distribution or GED are rejected in the LRuc tests. The phenomenon represents the failure rates in the normal distribution and GED, because the VaRs are statistically higher than the specific probability of the model. For example, in the normal distribution, the estimated failure rates are 0.0605, 0.0250, and 0.0183, which are statistically much higher than the specific probabilities of 0.05, 0.01, and 0.005. The same results appear in the GED, where the estimated failure rates are 0.0622, 0.0177, and 0.0083, which significantly exceed the specific probabilities of 0.05, 0.01, and 0.005. Identical results are shown in the AQLF and UL: the values are the lowest with the SGT distribution irrespective of the confidence level (95%, 99%, and 99.5%). With regard to the futures market, though the estimated failure rates either with the normal distribution or GED are fine in 95% VaR, the failure rates are significantly biased in higher confidence levels (99% and 99.5% VaR). Let us take the normal distribution, for example. The estimated failure rates are 0.0216 and 0.0161 for 99% and 99.5%

confidence levels, respectively; these values significantly exceed the specific probabilities of 0.01 and 0.005, respectively. In comparison, the estimated failure rates with the SGT distribution pass the coverage rate test in all the confidence levels. Similar results can be found in the AQLF and UL, and the values with both the normal distribution and GED are relatively higher than those with the SGT distribution.

We illustrate the time-varying scaling parameters in Figure 4. In the part indicating the GED in the figure, we see that the fat-tail parameter (κ) fluctuates around 2 prior to August 2007; moreover, there is a clear decline in the parameter in the global financial crisis period. Specifically, in the GED, the fat-tail is more apparent in the financial crisis period, but not so much in the other periods. In comparison, the scaling parameters of the SGT distribution in Part B of Figure 4 show that the skewness parameter λ is smooth around 0, indicating that the skewness is not very significant. However, two kurtosis parameters (κ and n) perform differently in the forecasting period. The first parameter, κ, is quite stable in the whole period and the average value is around 2, whereas the second parameter, n, is relatively volatile as compared to parameter κ. Except for the beginning of 2003 and mid-2007, the value of parameter n is low, indicating that the kurtosis exists significantly. In comparison, the scaling parameters in the SGT distribution can appropriately capture the volatility of gold and they show that the unexpected losses are smaller in the SGT distribution. Figure 5 shows the results of the forecasted VaR. Focusing on the latest period of the global financial crisis, we can easily observe that the forecasted VaR with the normal distribution and GED cannot capture the situation of high volatility, and the forecasted loss increases. However, the forecasted VaR with the SGT distribution is apparently different: the forecasted errors are relatively much smaller than the alternative distributions.

In sum, the forecasted VaR with the SGT distribution provides the most accurate out-of-sample forecasts either in the crude oil or gold markets. In the crude oil market, though all the distributions provide the correct coverage rate, the forecasted oil spot VaR with the normal distribution is only appropriate for the low confidence level, and the accuracy and performance distributions deteriorate and lose accuracy for the higher confidence levels. In comparison, the SGT distribution provides the most accurate out-of-sample forecasts within the strict VaR confidence levels. With regard to the gold market, the most appropriate distribution for the forecasted VaR is with the SGT distribution, and the failure rates in the normal distribution and GED for the VaR are statistically higher than the specific probability of the model. Therefore, the precise forecasting is the most important reason for adopting the SGT distribution.

CONCLUSION

This paper provides a comprehensive analysis using the flexible SGT distribution for modeling commodity volatilities and analyzing the time-varying scaling parameters, including those in the crude oil and gold markets. It also estimates the VaR within the framework of the GARCH-SGT model. The out-of-sample forecasting period covers a long period, including the most unsteady period of the global financial crisis. The empirical results show that the forecasted VaR with the SGT distribution provides the most accurate out-of-sample forecasts either in the crude oil or gold markets. In the crude oil market, though all the distributions provide the correct coverage rate, the forecasted oil spot VaR with the normal distribution is appropriate only for the low confidence level, and the accuracy and performance distributions deteriorate and lose accuracy for the higher confidence levels. In comparison, the SGT distribution provides the most accurate out-of-sample forecasts within the strict VaR confidence levels. Further, the time-varying parameters in the SGT distribution show that two kurtosis parameters (κ and n) perform differently in the forecasting period. The peakness parameter is close to that of the normal distribution (i.e., 2), indicating no peakness for the empirical distribution. However, the fat-tail characteristic significantly exists for the empirical distribution of returns. Focusing on the period of the current global financial crisis, we see that the estimation results of the scaling parameters in the GED and SGT distribution are totally different: the SGT distribution allows a very diverse level of skewness and kurtosis, and can capture the volatility more effectively. This is why the forecasting performance with the normal distribution and GED is not better than with the SGT distribution.

With regard to the gold markets, the most appropriate distribution for the forecasted VaR is the SGT distribution, and the failure rates in the normal distribution and GED for the VaR are statistically higher than the specific probability of the model. Moreover, the time-varying scaling parameters are similar to crude oil returns. The skewness parameter is close to 0, indicating that the skewness is not very significant; the peakness parameter is close to that of the normal distribution, indicating no peakness for the empirical distribution; and finally, the fat-tail parameter is small, indicating that the kurtosis significantly exists. Comparatively, the scaling parameters in the SGT distribution can capture the volatilities of gold effectively and they show that the unexpected losses are smaller in the SGT distribution. We focused on the latest period of the global financial crisis and found that the forecasted VaR with the normal distribution and GED are biased, whereas the SGT distribution can model the high volatility well. Finally, the estimated VaR within the SGT model is significantly superior to the other distributions in the crude oil and gold markets.

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Table 1. The Special Cases of SGT distributions

	λ	κ	n	Notes:	
Skew generalized t (SGT)	Free	Free	Free	λ > 0	skew to the right
Skew t (ST)	Free	2	Free	λ < 0	skew to the left
Skew GED (SGED)	Free	Free	∞		
Skew Normal	Free	2	∞	κ > 2	thinner tail than normal
Skew Laplace	Free	1	∞	κ < 2	thicker tail than normal
General t (GT)	0	Free	Free		
Student t	0	2	Free		
GED	0	Free	∞		
Normal	0	2	∞		
Laplace	0	1	∞		
Uniform	0	∞	∞		

Table 2. Descriptive Statistics

	Mean	S.D.	Skewness	Excess Kurtosis	J-B test
Part A. Crude oil					
Spot price	49.9529	26.3458	1.2846**	1.2981**	795.8244**
return	0.011	2.661	-0.2766**	4.2950**	1801.1006**
Futures price	49.9703	26.3638	1.2793**	1.2867**	787.7146**
return	0.0276	2.6552	-0.2082**	4.0639**	1602.8249**
Part B. Gold					
Spot price	484.8145	208.9687	0.8346**	-0.5371**	295.0343**
return	0.0518	1.8027	-0.2259**	23.0336**	50930.0510**
Futures price	486.2587	209.9169	0.8247**	-0.5695**	292.1880**
return	0.0511	1.2118	0.0956*	5.0259**	2427.3922**

Notes: J-B test is Jarque-Bera normality test. **and * represent significance under 1% and 10% level.

Table 3. Out-of-sample performance of alternative distributions for crude oil

Part A. Spot

		Mean	S.D.	LRuc	ABLF	AQLF	UL
Normal	95% VaR	-3.9992	1.7264	0.0464	0.0511	**0.2385**	**-0.0633**
	99% VaR	-5.7055	2.4509	2.4527	0.0138	0.0785	-0.0185
	99.5% VaR	-6.3302	2.7162	1.5697	0.0072	0.0542	-0.0126
GED	95% VaR	-3.9632	1.6979	0.0000	0.0500	0.2626	-0.0667
	99% VaR	-6.0390	2.5028	0.2166	0.0111	0.0699	-0.0172
	99.5% VaR	-6.8477	2.8146	1.5697	0.0072	0.0445	-0.0110
SGT	95% VaR	-3.9120	1.6634	0.8054	0.0546	0.2616	-0.0681
	99% VaR	-6.1914	2.4670	0.0485	0.0094	**0.0597**	**-0.0139**
	99.5% VaR	-7.2397	2.8140	0.1169	0.0055	**0.0362**	**-0.0077**

Part B. Futures

		Mean	S.D.	LRuc	ABLF	AQLF	UL
Normal	95% VaR	-3.9342	1.6231	0.1041	0.0516	0.1878	-0.0572
	99% VaR	-5.6043	2.3048	0.8384	0.0122	0.0482	-0.0130
	99.5% VaR	-6.2157	2.5544	0.4169	0.0061	0.0293	-0.0082
GED	95% VaR	-3.9322	1.6056	0.4124	0.0533	0.1885	-0.0567
	99% VaR	-5.8702	2.3483	0.5354	0.0083	0.0368	-0.0101
	99.5% VaR	-6.6131	2.6327	0.4838	0.0038	0.0199	-0.0061
SGT	95% VaR	-3.9221	1.5924	0.0130	0.0494	**0.1855**	**-0.0565**
	99% VaR	-6.0281	2.3227	2.1626	0.0067	**0.0283**	**-0.0092**

		Mean	S.D.	LRuc	ABLF	AQLF	UL
	99.5% VaR	-6.9432	2.6341	0.0957	0.0044	**0.0119**	**-0.0041**

Notes: * represents significance under 1% level. LRuc is the Log-likelihood test for correct unconditional coverage. ABLF is the average binary loss function. AQLF is the average quadratic loss function. UL denotes the unexpected loss, which refers to the average dollar loss caused by the failures of VaR model.

Table 4. Out-of-sample performance of alternative distributions for gold

Part A. Spot

		Mean	S.D.	LRuc	ABLF	AQLF	UL
Normal	95% VaR	-2.5769	1.4962	3.9672*	0.0605	0.3658	-0.0733
	99% VaR	-3.6759	2.1151	28.8773**	0.0250	0.1984	-0.0322
	99.5% VaR	-4.0783	2.3416	38.0757**	0.0183	0.1651	-0.0244
GED	95% VaR	-2.4878	1.3122	5.2706**	0.0622	0.3950	-0.0792
	99% VaR	-4.2121	2.1530	8.9335**	0.0177	0.1604	-0.0231
	99.5% VaR	-4.9446	2.5072	3.3448**	0.0083	0.1187	-0.0146
SGT	95% VaR	-2.4837	1.1153	2.4477	0.0587	**0.4427**	**-0.0804**
	99% VaR	-4.5451	1.9566	0.9358	0.0125	**0.1669**	**-0.0211**
	99.5% VaR	-5.6366	2.4104	0.5503	0.0037	**0.1078**	**-0.0130**

Part B. Futures

		Mean	S.D.	LRuc	ABLF	AQLF	UL
Normal	95% VaR	-1.8894	0.6966	2.1878	0.0577	0.1329	-0.0459
	99% VaR	-2.6959	0.9880	18.5572**	0.0216	0.0500	-0.0182
	99.5% VaR	-2.9911	1.0947	28.0883**	0.0161	0.0355	-0.0133
GED	95% VaR	-1.8807	0.7054	2.1878	0.0577	0.1333	-0.0461
	99% VaR	-2.9817	1.1008	5.7296*	0.0161	0.0368	-0.0137
	99.5% VaR	-3.4252	1.2600	8.4500**	0.0105	0.0223	-0.0085
SGT	95% VaR	-1.9780	0.8220	0.1156	0.0518	**0.1256**	**-0.0438**
	99% VaR	-3.3187	1.4320	1.8863	0.0135	**0.0311**	**-0.0115**
	99.5% VaR	-3.9548	1.7703	2.5151	0.0080	**0.0171**	**-0.0061**

Notes: ** and * represent significance under 1% and 5% level. LRuc is the Log-likelihood test for correct unconditional coverage. ABLF is the average binary loss function. AQLF is the average quadratic loss function. UL denotes the unexpected loss, which refers to the average dollar loss caused by the failures of VaR model.

Table 5. Descriptive statistics of time-varying scaling parameters

			Mean	S.D.	Min.	Max.
Part A. Crude oil						
Spot:	GED ~ kurtosis:	κ	1.315	0.198	0.916	1.779
	SGT ~ kurtosis:	κ	2.347	0.335	1.566	4.371
		n	9.993	14.033	3.732	183.453
	skew:	λ	-0.049	0.044	-0.151	0.101
Futures:	GED ~ kurtosis:	κ	1.221	0.179	0.913	1.585
	SGT ~ kurtosis:	κ	2.204	0.388	1.444	3.742
		n	87.476	121.474	3.770	363.980
	skew:	λ	-0.054	0.055	-0.160	0.132
Part B. Gold						
Spot:	GED ~ kurtosis:	κ	1.898	0.147	1.567	2.289
	SGT ~ kurtosis:	κ	1.934	0.527	1.062	5.294
		n	4.994	3.405	2.048	70.813
	skew:	λ	-0.094	0.081	-0.285	0.080
Futures:	GED ~ kurtosis:	κ	1.531	0.239	1.168	2.244

SGT ~ kurtosis:	κ	2.001	0.574	1.065	5.479
	n	31.372	68.244	2.001	290.703
skew:	λ	-0.127	0.126	-2.183	0.165

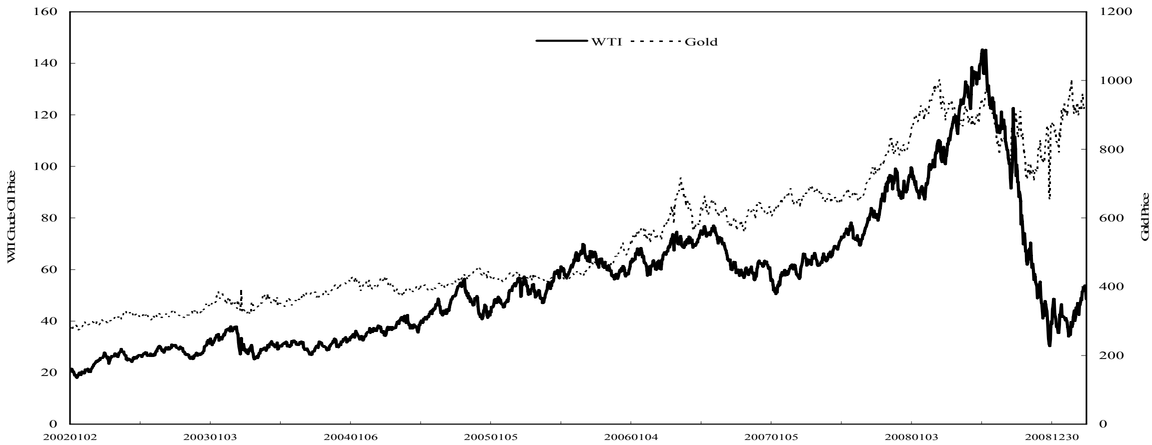
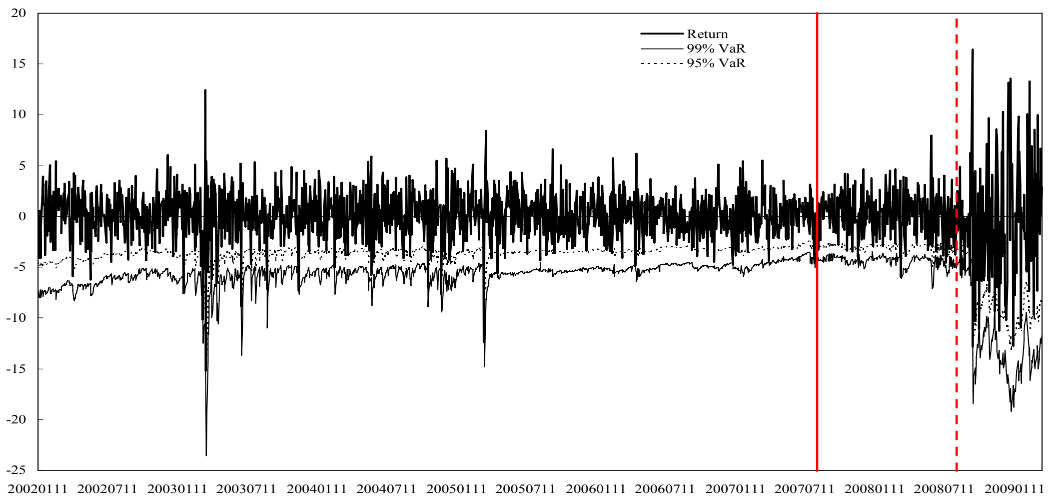
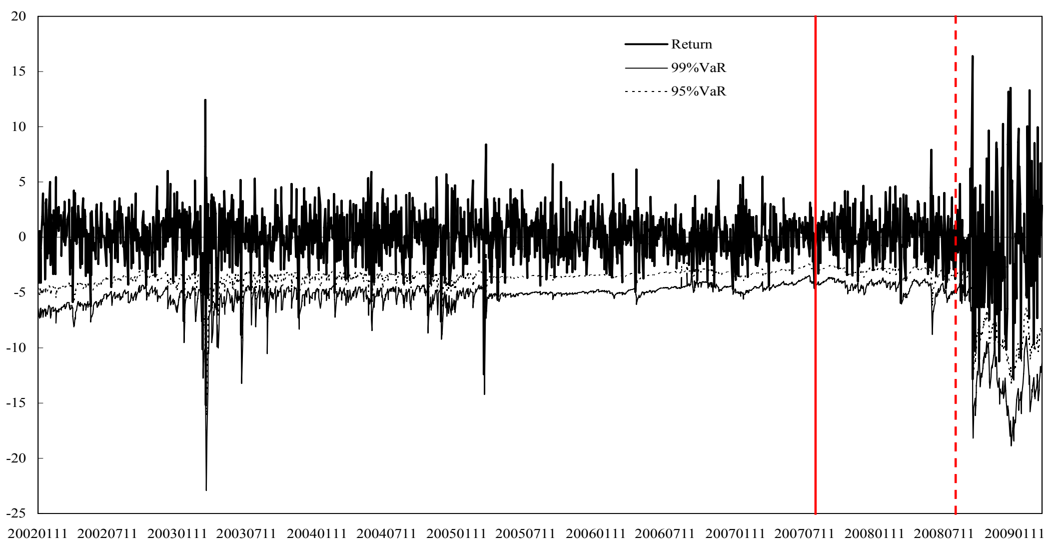


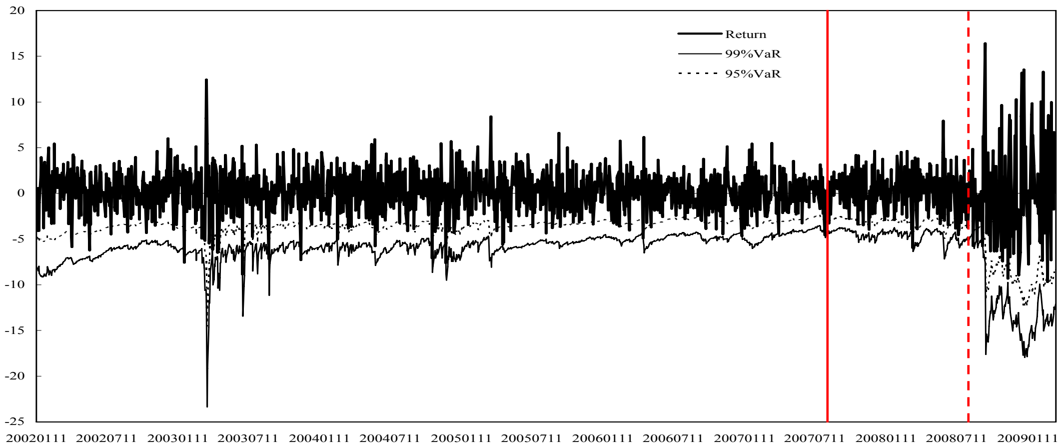
Figure 1. The time series plot of crude oil and gold



Part A. GED distributions

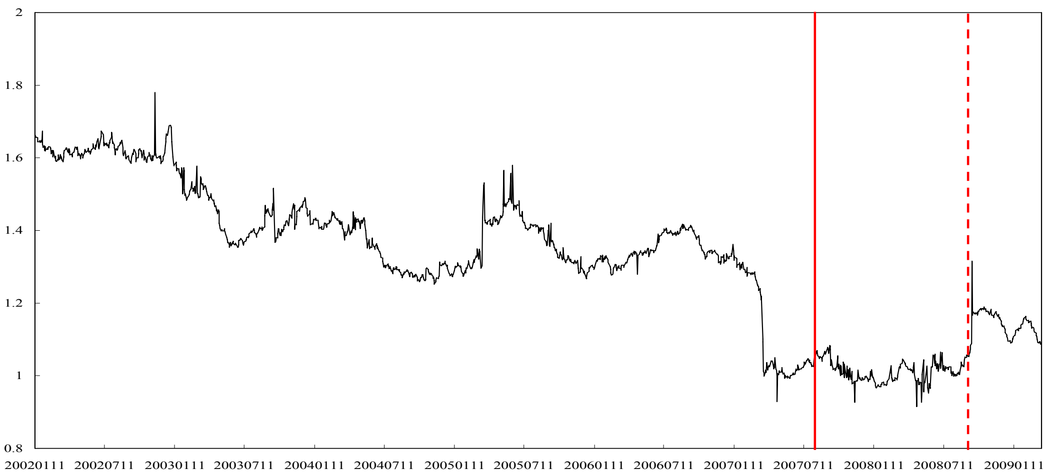


Part B. Normal distributions

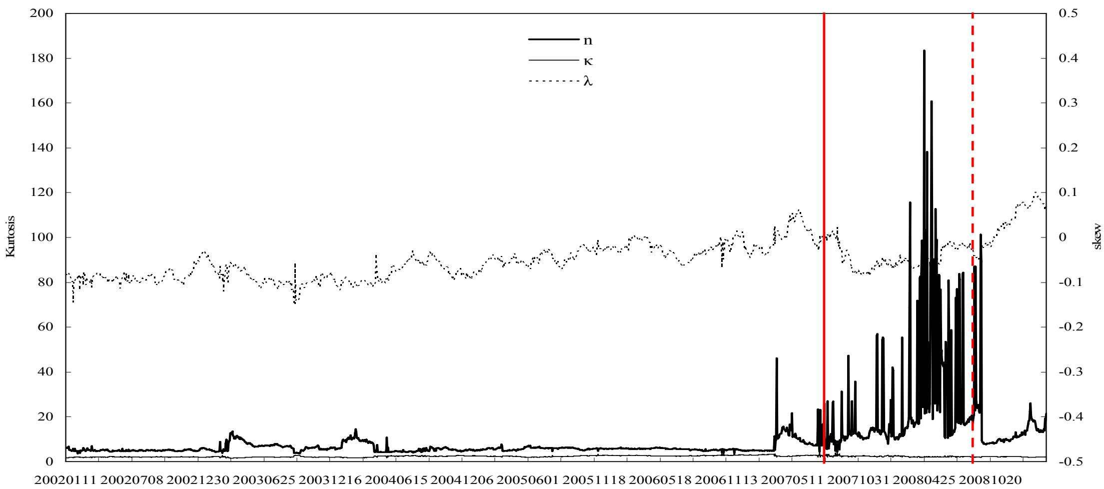


Part C. SGT distribution

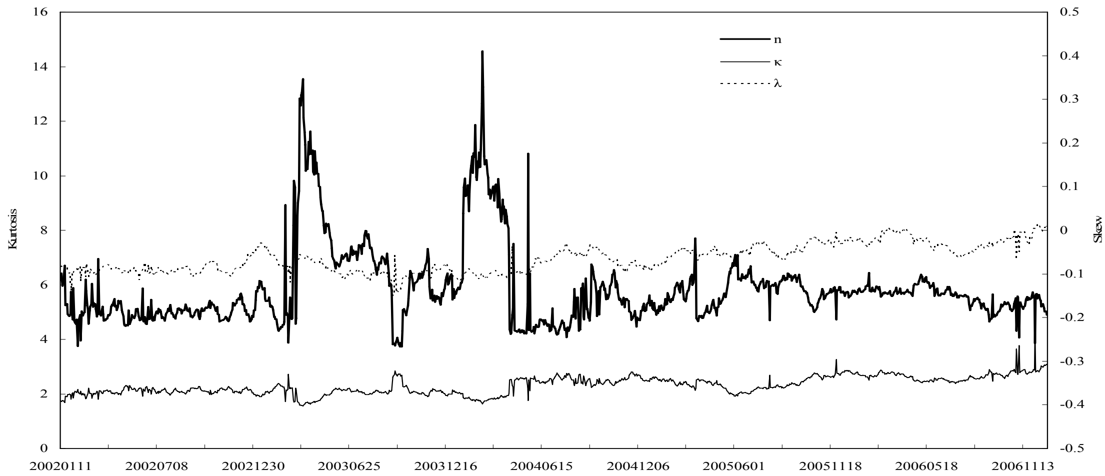
Figure 2. Forecasted VaR with different distributions in crude oil spot



Part A. Kurtosis parameter in the GED distribution

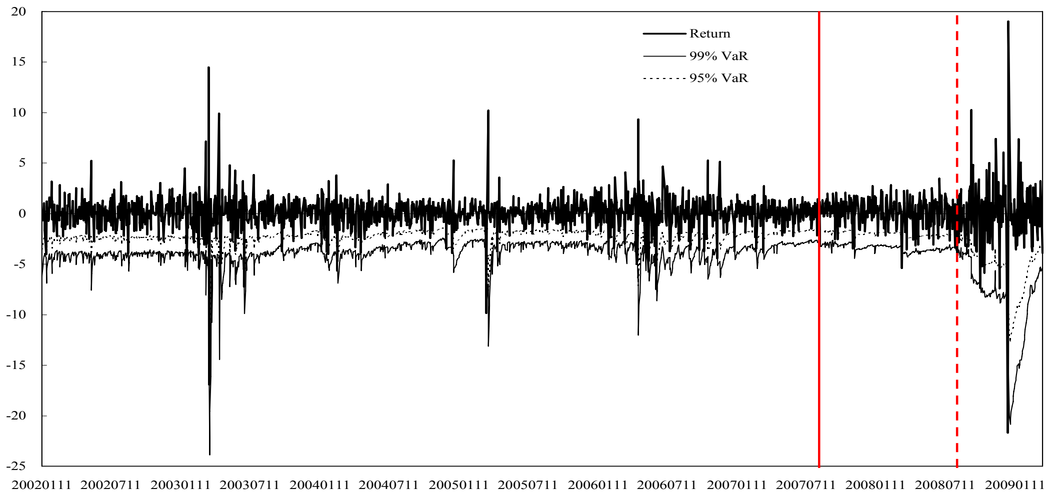


Part B. Skewness and kurtosis parameters in the SGT distribution

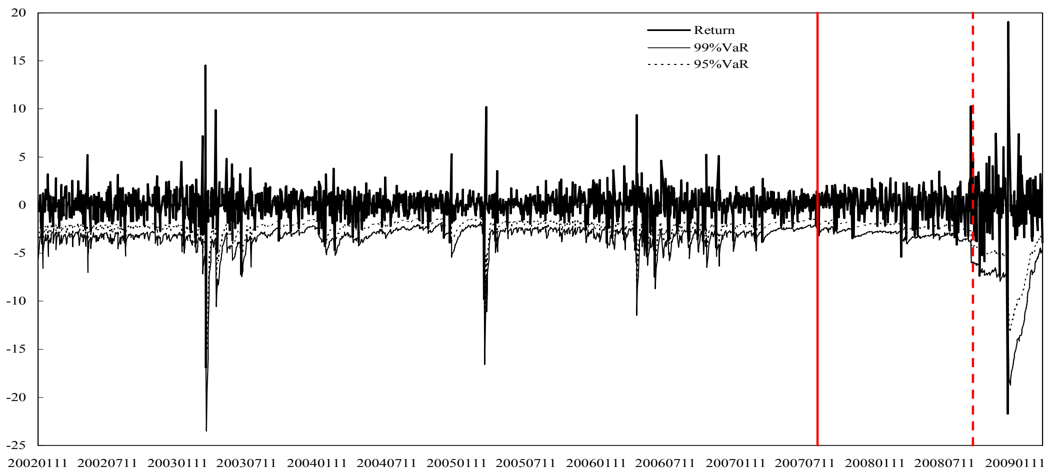


Part C. Skewness and kurtosis parameters in the SGT distribution exclude the period of global financial crisis

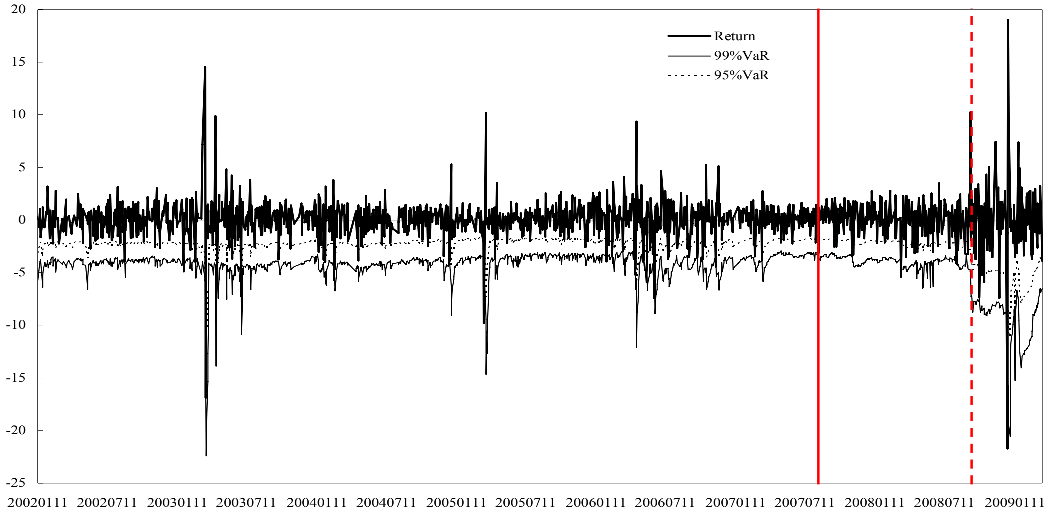
Figure 3. The time-varying scaling parameters



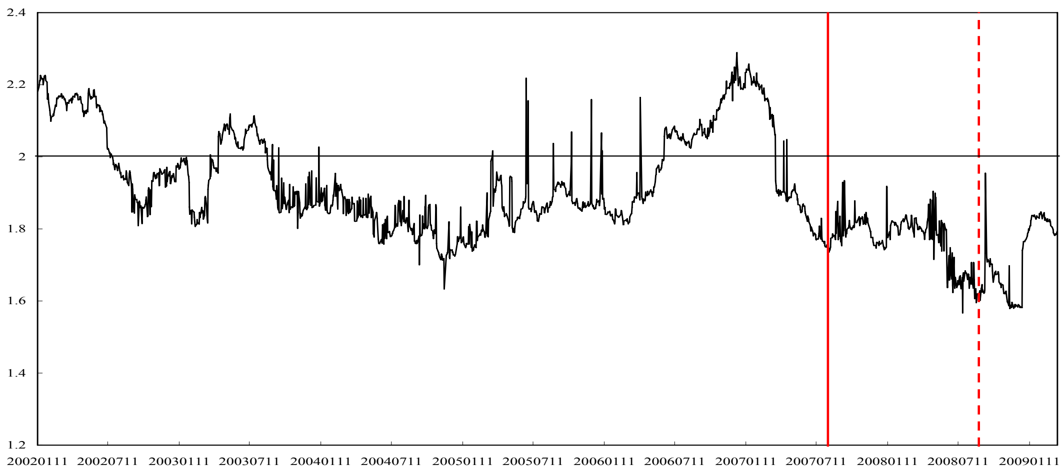
Part A. GED distribution



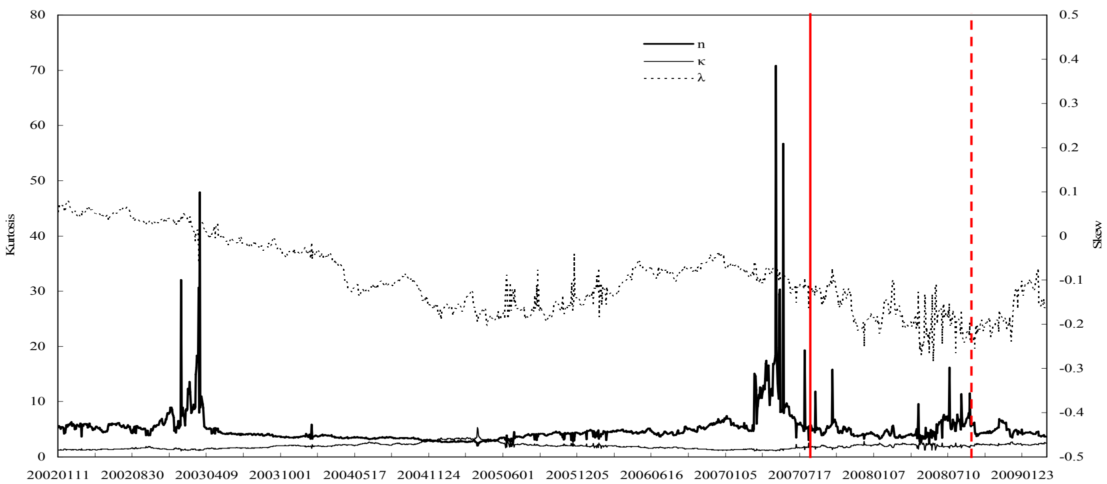
Part B. Normal distribution



Part C. SGT distribution
Figure 4. Forecasted VaR with different distributions in gold spot



Part A. Kurtosis parameters in GED distribution



Part B. Skewness and kurtosis parameters in the SGT distribution
Figure 5. The time-varying scaling parameters