1. Please compute

   a. $\int x\sqrt{2x+3}\,dx$ (5%)

   b. $\int_0^1 x^2 \ln x dx$ (5%)

   c. $\lim_{h\to 0} \frac{1}{h}\int_0^h \sqrt{1+t+t^2}dt$ (5%)

   d. $\frac{d}{dx}[\int_1^{\tan x}(1+t^2)dt$ (5%)

2. Please find the maximum or minimum of

   $f(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{1}{2}(\frac{x-\mu}{\sigma})^2)$ $-\infty < x < \infty$ (10%)

3. Please compute (10%)

   Max. (Min.) $f(x, y, z) = x^2 + y^2 + z^2$

   s.t. $xyz = -1$

   $x + y + z = 1$

4. Let $f(x, y) = \sqrt{xy}$, $D = \{(x, y) | x^2 \le y \le \sqrt{x}, x \in [0,1]\}$, please find

   $\iint_D f(x, y)dxdy$ (10%)

5. Please describe and explain following terms:

   a. Chebyshev's inequality (5%)

   b. Central Limit Theorem (5%)

6. Let X, Y be two random variables with joint pdf (probability density function) given by

   $f(x, y) = 6x$ $0 < x < y < 1$

   Find the conditional pdf of X and Y respectively. (10%)

7. Let $X_1, X_2, \ldots X_n$ be a random sample from a normal distribution with mean μ and variance σ^2, if

   $$\hat{\mu} = \frac{2}{n(n+1)}(X_1 + 2X_2 + \ldots + nX_n)$$

   Please show that $\hat{\mu}$ is an unbiased estimator of μ. (10%)

接 次 頁

8. Let $X_1, X_2, \ldots X_n$ be iid with pdf

$f(x|\theta) = \theta\, x^{\theta-1}$, $0 \le x \le 1, 0 < \theta < \infty$.

Find the MLE of θ and show that its variance $\to 0$ as $n \to \infty$. (10%)

9. To investigate the relationship between X (tenure) and Y (income) by interviewing 25 employees, we get

$\sum X = 100$, $\sum Y = 2000$, $\sum X^2 = 510$, $\sum Y^2 = 187{,}280$,

$\sum XY = 9{,}650$

Please find out the linear regression equation and correlation coefficient. (10%)