

南華大學九十六學年度 碩士班 招生考試試題卷

系所別：環境管理研究所

科目編號：1102-1

科目：微積分

試題紙第 頁共 頁

1. Find y' (30%)

(a) $y = \sqrt{\tan^3(x^4) + x^2}$

(b) $y = \cos x \cot x \csc x$

(c) $y = \frac{(x^2 + 1)(x^3 + 1)}{x^{1/2}}$

(d) $\ln\left(\frac{x}{y}\right) = x + y$

(e) $y = (\ln x)^{\log_2 x}$

(f) $y = 2^{\log_2 e^x}$

2. Determine values of a and b that make the given function continuous. (4%)

$$f(x) = \begin{cases} 2\frac{\sin x}{x}, & \text{if } x < 0 \\ a, & \text{if } x = 0 \\ b \cos x & \text{if } x > 0 \end{cases}$$

3. Determine all vertical and horizontal asymptotes. (6%)

$$f(x) = \begin{cases} \frac{4x}{x-4}, & \text{if } x < 0 \\ \frac{x^2}{x-2}, & \text{if } 0 \leq x < 4 \\ \frac{e^{-x}}{x+1}, & \text{if } x \geq 4 \end{cases}$$

4. If $f(x) = \cos(2x + 1)$, find $f^{(103)}(x)$. (5%)

5. Find the indicated limits (10%)

(a) $\lim_{x \rightarrow \infty} (\ln x - x)$,

(b) $\lim_{x \rightarrow \infty} \left(\frac{1}{x}\right)^{\frac{1}{x}}$.

6. Evaluate the integrals (25%)

(a) $\int \frac{e^{1/x}}{x^2} dx$

(b) $\int \ln(2x + 1) dx$

(c) $\int \sin^4 x \cos^5 x dx$

(d) $\int \frac{ax}{x^2 - bx} dx$

(e) $\int \frac{1}{\sqrt{x} - \sqrt[3]{x}} dx$ (Hint: $u = \sqrt[6]{x}$)

7. Prove that (3%)

$$\int (\ln x)^n dx = x(\ln x)^n - n \int (\ln x)^{n-1} dx$$

8. Sketch the curve $r = 3(1 + \cos \theta)$ in polar coordinate system and find the area that it enclosed. (8%)

9. Locate and classify all critical points for $f(x, y) = x^3 + x^2 - xy + y^2 + 4$. (4%)

10. Find the Taylor series of the given function $f(x) = \ln x$ at $x = 1$. (5%)