

南華大學九十五學年度學士班暨進修學士班
轉學生招生考試考試試題卷

招生
學系

學士班 2 年級資訊工程學系

編號：D2-28-11

科目：微積分

試題紙第 頁共 頁

1. Find the derivative $\frac{dy}{dx}$. (20%)

(a) $y = \frac{v^3 - 2v\sqrt{v}}{v}$

(b) $y = \sin(\tan \sqrt{\sin x})$

(c) $xy = \cot(xy)$

(d) $y = (\ln x)^{\cos x}$

2. Find the n^{th} derivative $f^{(n)}(x)$ of the function $f(x) = \frac{1}{x+1}$. (5%)

3. Suppose that f is continuous on $[0, 4]$, $f(0) = 1$ and $2 \leq f'(x) \leq 5$ for all x in $(0, 4)$, show that $9 \leq f(4) \leq 21$. (hint: use the mean value theorem) (5%)

4. Evaluate the integral (30%)

(a) $\int x^2(x^3 + 5)^9 dx$

(b) $\int \sec \theta d\theta$

(c) $\int \sin^4 x \cos^5 x dx$

(d) $\int \theta \tan^2 \theta d\theta$

(e) $\int \frac{1}{(x+a)(x+b)} dx$

(f) $\int_0^1 \ln x dx$

5. For the cardioid $r = 1 + \sin \theta$, (a) find the slope of tangent line when $\theta = \pi/3$, and (b) find the points on the cardioid where the tangent line is horizontal or vertical. (10%)

6. Show that the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$. (5%)

7. If p is a positive real number, prove that the series (10%)

$$\sum_{i=1}^{\infty} \frac{1}{i^p} \begin{cases} \text{is convergent if } p > 1 \\ \text{is divergent if } p \leq 1 \end{cases}$$

8. Find the Taylor series expansion of the given functions $f(x) = xe^x$ at $x = 0$. (5%)

9. If $z = e^{u^2+v^2}$, $u = \frac{x}{y}$, $v = x^2y$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $(x, y) = (2, 1)$. (10%)