在信用交易下考慮非瞬間收到及允許缺貨的存貨模型

An Inventory Model with Noninstantaneous Receipt and Allowable Shortages under Trade Credit

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Abstract

It is customary in inventory theory to assume that payments must be paid to the supplier for the items immediately after receiving the consignment. In real business transactions, the supplier allows a certain fixed credit period to settle the account for stimulating retailer's demand and increasing revenue. From the retailer's viewpoint, during the credit period before payment must be made, he/she can sell the items and continue to accumulate revenue and earn interest. This article explores an inventory system with noninstantaneous receipt and allowable shortages under conditions of permissible delay in payments. The noninstantaneous receipt model under trade credit is newly introduced. The purpose of this article is to determine the optimal replenishment policies under conditions of noninstantaneous receipt and trade credit. We present an algorithm to find the optimal solutions so that the total relevant cost per unit time is minimized. Numerical examples are presented to illustrate the proposed model.

Keywords**:** inventory, noninstantaneous receipt, trade credit

1. Introduction

Trade credit is increasingly important payment behavior in real business transactions. However, traditional inventory strategies, it is almost concentrated on solving the optimal order quantity and reorder point but ignoring the type of payment. In practice, the supplier allows a certain fixed credit period to settle the account for stimulating retailer's demand. From the retailer's viewpoint, during the credit period before payment must be made, he/she can sell the items and continue to accumulate revenue and earn interest. Goyal (1985) first develops an inventory model under the condition of permissible delay in payments. Later, Aggarwal and Jaggi (1995) extend Goyal (1985) model to consider an inventory model of deteriorating items with permissible delay in payments. Next, Jamal *et al*. (1997) further generalized the model to allow for shortages. Other authors also considered similar issues relating to delay in payments (see, e.g., Chu *et al*. (1998), Chung (1998), Sarker *et al*. (2000a, 2000b), Liao *et al*. (2000), Chang and Dye (2001), Teng (2002) and Salameh *et al*. (2003)).

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All of the above models, they assumed that an entire order is received into inventory at one time (i.e., infinite replenishment rate). In empirical observations, the order quantity is frequently received gradually over time and the inventory level is depleted at the same time it is being replenished. This version of the economic order quantity (EOQ) model is known as the noninstantaneous receipt (i.e., finite replenishment rate) model. The "Noninstantaneous Receipt" and economic production quantity (EPQ) are the same in mathematical model. However, there is slight difference in managerial implication. The "Noninstantaneous Receipt" means that the retailer receives the order quantity and sales at the same time in the retailer business. The EPQ means that the manufacturer produces the products and sales at the same time in the manufacturing industry. The concept of "Noninstantaneous Receipt" can be observed in Stevenson (1996, p.542) and Taylor III (1999, p.786).

For generality, this study develops an inventory model with noninstantaneous receipt and allowable shortages under trade credit. We provide an easy-to-use algorithm to find the optimal order strategies. Also, several numerical examples for illustration the theoretical results. The rest of this paper is organized as follows. In Section 2, we describe the notation and assumptions used throughout this study. In Section 3, the model is mathematically formulated. In Section 4, an algorithm is established to determine the replenishment policies. Numerical examples are provided in Section 5 to illustrate the results. Finally, we draw the conclusions and the future research in Section 6.

2. Notation and assumptions

First of all, the following notation and assumptions are employed throughout this paper so as to develop the inventory model.

Notation

t_1 = point of time when the backordered demand is completely satisfied from the replenishment.

t_2 = point of time when inventory level is maximum and replenishment terminates.

t_3 = point of time when all inventory is consumed.

t_4 = point of time when shortage level is maximum.

c_1 = ordering cost per order.

c_2 = unit holding cost per unit time excluding interest charges.

c_3 = unit shortage cost per unit time.

c = item cost ($/unit).

p = selling price ($/unit), with $p > c$.

I_c = interest charged per dollar per unit time.

I_d = interest earned per dollar per unit time.

M = the last time of permissible delay in settling the accounts.

K = per unit time rate at which the order is received over time, i.e., replenishment rate.

D = per unit time rate at which inventory is demanded, i.e., demand rate.

Assumptions

(1) The replenishment rate K is finite and greater than the demand rate D i.e., $K > D$.

(2) Retailer would not consider paying the payment until receiving all items.

(3) Supplier offers a certain fixed credit period M to settle the account.

(4) During the time the account is not settled, generated sales revenue is deposited in an interest bearing account. At the end of this period, the retailer starts paying for the interest charges on the items in stocks.

(5) Shortages are allowed.

3. Model formulation

The total relevant cost per cycle consists of the following elements, which can be easily obtained by using Figure 1 and trigonometry.

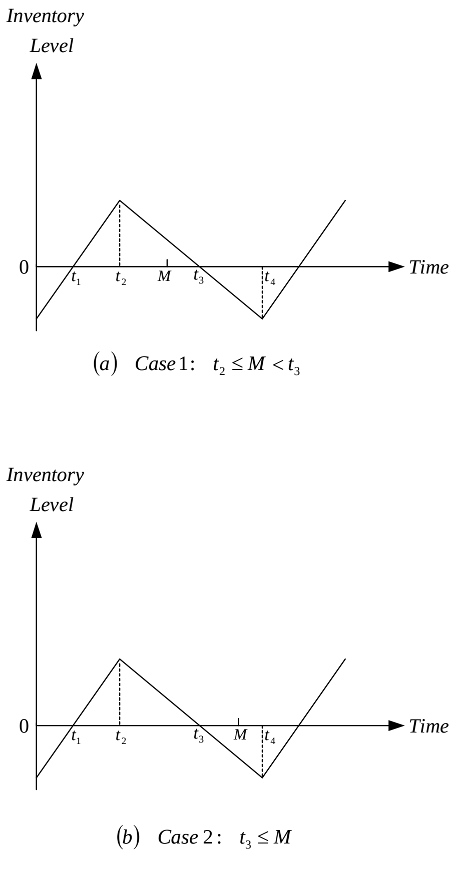

Figure 1 Inventory system

(i) Ordering cost per order $= c_1$. (1)

(ii) Holding cost per cycle $= \frac{c_2}{2}(K-D)(t_2-t_1)(t_3-t_1)$. (2)

(iii) Shortages cost per cycle $= \frac{c_3}{2}(K-D)t_1(t_1+t_4-t_3)$. (3)

(iv) The interest payable per cycle for two case is given as follows:

Case 1: $t_2 \le M < t_3$ (see Figure 1 (a))

$$\text{Interest payable per cycle} = \frac{c I_c D(t_3-M)^2}{2}. \tag{4}$$

Case 2: $t_3 \le M$ (see Figure 1 (b))

In this case, there is no interest payable.

(v) During the time the account is not settled, generated sales revenue is deposited in an interest bearing account. Hence, the interest earned per cycle for two case is given as follows:

Case 1: $t_2 \le M < t_3$ (see Figure 1 (a))

$$\text{Interest earned per cycle} = \frac{p I_d D M^2}{2}. \tag{5}$$

Case 2: $t_3 \le M$ (see Figure 1 (b))

$$\text{Interest earned per cycle} = p I_d\left[\frac{D t_3^2}{2} + D t_3(M-t_3)\right]. \tag{6}$$

Therefore, the total relevant cost per unit time for each case is obtained as follows:

$$TC_1 = \frac{1}{t_4}\left\{c_1 + \frac{c_2}{2}(K-D)(t_2-t_1)(t_3-t_1) + \frac{c_3}{2}(K-D)t_1(t_1+t_4-t_3) + \frac{c I_c D(t_3-M)^2}{2} - \frac{p I_d D M^2}{2}\right\}. \tag{7}$$

and

$$TC_2 = \frac{1}{t_4}\left\{c_1 + \frac{c_2}{2}(K-D)(t_2-t_1)(t_3-t_1) + \frac{c_3}{2}(K-D)t_1(t_1+t_4-t_3) - p I_d\left[\frac{D t_3^2}{2} + D t_3(M-t_3)\right]\right\}. \tag{8}$$

The inventory level is maximum at t_2 and the shortage level is maximum at t_4. It gets

$$(K-D)(t_2-t_1) = D(t_3-t_2) \text{ and } (K-D)t_1 = D(t_4-t_3). \tag{9}$$

That is

$$t_2 = \frac{Kt_1 - Dt_1 + Dt_3}{K} \quad \text{and} \quad t_4 = \frac{(K-D)t_1}{D} + t_3. \tag{10}$$

Hence, the total relevant cost per unit time is a function of two variables t_1 and t_3 because of (10). The necessary condition for TC_1 to be minimum is the optimal solution satisfies:

$$\frac{\partial TC_1(t_1, t_3)}{\partial t_1} = 0 \quad \text{and} \quad \frac{\partial TC_1(t_1, t_3)}{\partial t_3} = 0, \tag{11}$$

simultaneously. We denoted that the optimal solution (t_1^a, t_3^a) satisfies the second-order sufficient conditions for a minimum value.

Similarly, The necessary condition for TC_2 to be minimum is the optimal solution satisfies:

$$\frac{\partial TC_2(t_1, t_3)}{\partial t_1} = 0 \quad \text{and} \quad \frac{\partial TC_2(t_1, t_3)}{\partial t_3} = 0, \tag{12}$$

simultaneously. We denoted that the optimal solution (t_1^b, t_3^b) satisfies the second-order sufficient conditions for a minimum value.

4. Solution procedure

The optimal replenishment policies and minimum total cost per unit time can be obtained by using the following algorithm:

Algorithm 1.

Step 1. Determine t_1^a and t_3^a from (11). Substituting (t_1^a, t_3^a) into (10), we can find (t_2^a, t_4^a). If $t_2^a \le M < t_3^a$, obtain $TC_1(t_1^a, t_2^a, t_3^a, t_4^a)$ from (7); otherwise (t_1^a, t_3^a) is infeasible.

Step 2. Determine t_1^b and t_3^b from (12). Substituting (t_1^b, t_3^b) into (10), we can find (t_2^b, t_4^b). If $t_3^b \le M$, obtain $TC_2(t_1^b, t_2^b, t_3^b, t_4^b)$ from (8); otherwise (t_1^b, t_3^b) is infeasible.

Step 3. By comparing $TC_1(t_1^a, t_2^a, t_3^a, t_4^a)$ and $TC_2(t_1^b, t_2^b, t_3^b, t_4^b)$, select the optimal solution (denoted by $(t_1^*, t_2^*, t_3^*, t_4^*)$) with the least total cost per unit time (denoted by TC^*).

5. Numerical examples

In order to illustrate the above solution procedure, let us consider an inventory system with the following data: K =3000 units/year, D =1000 units/year, $c_1 = \$90$ per setup, $c_2 = \$2$ /unit/year, c_3 =\$8/unit/year, c= \$20/unit, s = \$25/unit, $I_d = 0.13$/year and $I_c = 0.15$/year. The optimal solutions for different parameters values of M are shown in table 1. For instance, when $M = \frac{30}{365}$=0.0822 year, the optimal values t_1^*=0.0270 year, t_2^*=0.0812 year, t_3^*=0.1897 year, t_4^*=0.2437 year and TC^*=\$539.61. It implies that the retailer should pay the payment before all inventory is consumed.

Table 1. Optimal solutions

M	t_1^*	t_2^*	t_3^*	t_4^*	TC^*
30/365	0.0270	0.0812	0.1897	0.2437	539.61
45/360	0.0230	0.0798	0.1934	0.2393	437.46
60/360	0.0187	0.0777	0.1956	0.2329	329.64
75/360	0.0141	0.0750	0.1967	0.2250	215.72

6. Concluding remarks

In this study, we develop an inventory system with noninstantaneous receipt and allowable shortages under conditions of permissible delay in payments. The noninstantaneous receipt model under trade credit is newly introduced. The cash discount and noninstantaneous receipt are newly introduced. We provide an easy-to-use algorithm to find the optimal order strategies. In another word, if the supplier provides a permissible delay and an entire order is not received into inventory at one time, the retailer can use Algorithm 1 to get the optimal order strategies. Also, we can find that the results are consistent with the economic senses from table 1. When the supplier extends credit period, the retailer's total relevant cost per year decrease. In further research, we would like to consider the deteriorating items or variable demand rate (e.g., time-varying demand or stock-dependent demand).

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