A Mathematical Model of Control Pollutant Emissions with Tradable Quota-Permit Policy

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Abstract

The steady pace of economic growth not only worsened industrial pollution but also depleted the general living standards. Built upon the premises of total amount control, the dissertation begins by using the marketable permit policy, followed by tradable quota-permit policy by the policymaker. The findings concluded from this research yielded a recommendation that a static optimization model to come up with the requirement of total amount control. The study wraps up by proposing a static optimization model with tradable quota-permit policy, a practical model that not only closely simulates the actual scenario, but also fittingly addresses various measures in environmental protection management.

Keywords: policy, pollutant emissions, total amount control, mathematical model, static optimization model, marketable permit, tradable quota-permit

1. Introduction

Followed by the development of economics, the excessive consumption and industrial production have induced the damage on ecology, and also the decrease of life quality. Pollutant emissions are the by-product of industrial production and consumption, and must be controlled under the condition of not affecting economic development. Aiming at the pollutant emissions reduction of total amount control, the policymaker applies the tradable quota-permit policy for this purpose.

The marketable permit that Baumol & Oats (1998), Krupnick et al (1983) and Montgomery (1972) asserted were intended to minimize the total reduction costs on a specific pollutant coming from k types of fixed pollution sources within a region, assuming that the reduction cost of pollution source i would be taken as the function of the expected amount of emissions, $Ci(ei)$, shown in Matrix 1 below.

Matrix 1.

j' / source I		Receptors	Total emissions amount E
		1, 2, ... n'	
Fixed pollution source	1 2 . . . k	$d_{ij'}$	e_1 e_2 . . . e_k
Standard emissions concentration Q^s		$q_1^s, q_2^s, \ldots , q_{n'}^s$	mim .imize $\sum_i C_i(e_i)$

Where, $Q^s = (q_1^s, q_2^s, \ldots , q_{n'}^s)$, Q^s being the vector of total standard emissions concentration at receptors,

$q_{j'}^s$ being the total standard emissions concentration at j' receptor,

$i = 1, 2, \ldots , k$, i being the source of pollution,

$j' = 1, 2, \ldots , n'$, j' being the receptor,

$E = (e_1, e_2, \ldots , e_k)$;

E being the vector of total emissions amount at pollution sources,

e_i being the total emissions amount at i source,

$d_{ij'}$ being the transfer or diffusion coefficient of the i pollution source at j' receptor,

$D=[d_{ij'}]$, D being the transfer or diffusion coefficient matrix of k times n'.

A Model (1) will be derived from Matrix 1 and being expressed as, Objective function

$$Minimize \sum_{e_i} C_i(e_i) \tag{1}$$

$$\text{subject to} \quad E \bullet D \leq Q^s$$

$$E \geq 0$$

The objective function Model (1) is to decide the optimal pollution emissions (e^* or E^*) such that the total reduction costs, $\sum C_i(e_i)$, can be the minimal. It should be subjected to the upper limit of standard emissions concentration, $E \bullet D \leq Q^s$. Where $d_{ij'}$ times e_i would provide marketable pollution permit for pollution source i at receptor j'.

Since the permits allow the polluters (the pollution source) to conduct trade via market pricing function that provides effective distribution of the emissions amount sustainable to the environment. Hence a producer (the pollution source) that emits pollution finds out that he may be able to obtain a more efficient, or lower cost approach to minimize the emissions amount, such polluter will be able to sublet the excess of his quota to other polluters. Under the adjusted and redistributed scheme that allows negotiable pollution permits would help to minimize the total reduction costs of pollutant emissions (Wang, 1997).

This marketable pollution permit (APS) can be loosely defined as having a given $q_{j'}$ permits released to various receptors by a policymaker of environmental protection authorities, while such permits can then be traded through competitive bidding to arrive at an equilibrium solution. This solution also address the minimal total reduction costs with most efficient criteria (Krupnick, 1983) (Montgomery, 1972).

Model (1), as an integer-programming model, has a unique feature being that it is a mathematical programming model, such model is none but a variation typical of Knapsack problem within the domain of operations research.

Nevertheless, there are several obstacles for environmental protection authorities to tackle the APS using Model (1). First, authority has to be address in the permissible quota assigned to each control region, meaning that the feature of the emission feature $d_{ij'}$ for each control region has to be identical. Second, a complete solution is required for the cost minimization proposed in Model (1). Third, to obtain such complete solution, it not only requires locating the $d_{ij'}$ of the pollutant quality model, but also a complete emission inventory as well. Forth, there is also the difficulty to define total reduction costs function, $\sum_i C_i(e_i)$, and the method to solve the mathematical model. All of the above issues require consideration in the design of its practical implementation.

The proposed marketable quota-permit policy (QPS) is often adopted as a better one, whose objective is to cut down pollution from the source. Under the policy framework come certain basic quota-permit within each pollution source, and can be traded among the sources. The objective that environmental protection authorities (EPA) seek to accomplish is that once the policy is streamlined into implementation, the total reduction costs on fixed pollution source can be reduced to a minimal level. It allows the receptors, *j'*, to be regulated within the emissions standard, $q_{j'}^s$, assigned to the control region. Environmental protection authorities could begin by assigning a set quota-permit, in tonnage/annum expressed as e^0_i, of some sort of designated authorization to the existing fixed pollution source. In the meantime, each source to dispense a set charge on each unit of P_0 quota-permit used as part of the fund for pollution prevention measures sought by environmental protection authorities. EPA has its own individual cost of pollution prevention is collected from the sources, denoted as the reduction cost function, $C_i(e_i)$. The responsibility of environmental protection authorities approves working preventive measures sought by fixed pollution source that tackles the amount of pollutant emissions, thus when a specific business' quota-permit runs low, environmental protection authorities may also approve such source to trade their quota-permit difference, $e_i - e_i^0$, among the sources. The cost of the quota-permit is defined as a fixed price, P. No exception is to be allowed to go over the emission standard, $q_{j'}^s$, in terms of the basic quota-permit assigned by environmental protection authorities.

2. The Static QPS Model

To built a marketable or tradable quota-permit (QPS) under the proposed policy framework, Mathematical Model (2) that offers an optimized solution on the total amount control is denoted as

Objective function,

$$\underset{ei}{Min}\ Z(e_i) = \sum_i C_i(e_i) + P \cdot \sum_i (ei - e_i^0) + P_o \cdot \sum_i e_i^0 \qquad (2)$$

subject to

$$e_i \cdot d_{ij'} \leq q^s_{j'}\ ,\ or\ \ E \cdot D \leq Q^s$$

$$e_i^o \cdot d_{ij'} \leq q^s_{j'}\ ,\ or\ \ E^0 \cdot D \leq Q^s$$

$$e_i \geq 0\ ,\ e_i^0 \geq 0$$

The symbols used in the formula are defined as,

i : Fixed pollution source, i = 1, 2, ... , k.

j' : Receptor, or point of discharge, monitoring station, j'=1, 2, ... ,n'.

C_i : The reducing cost function of a fixed pollution source i

e_i : The total emission amount (ton/yr) of a fixed pollution source i

e^0_i : The basic quota-permit (ton/yr) of a fixed pollution source i

P : Market price of the quota-permit

$P0$: Quota-permit operating fee

$q^s_{j'}$: Standard emission concentration of a receptor j'

$d_{ij'}$: The transfer or diffusion coefficient of a fixed pollution source i at receptor j',

Which can also be expressed as shown in Matrix II,

Matrix II

I \ j'	1 2 … … … … n'	E	E^0
1	$d_{ij'}$	e_1	e_1^0
2		e_2	e_2^0
.		.	.
.		.	.
.		.	.
.		.	.
m		e_k	e_k^0
Q^s	$q^s_1, q^s_2, \dots\dots\dots\dots, q_n^s$	$E \cdot D \le Q^S$ $E^0 \cdot D \le Q^S$	$\underset{i}{\overset{\text{mim imize}}{\sum}} C_i(e_i)$

3. The Discussion of the QPS Model

In this paper, we prefer to name the Model (2) as static marketable or tradable quota-permit (QPS) Model. Due to the QPS Model does not happen with time, it is not a dynamic, so it is a static QPS Model.

3.1 The characteristics of QPS Model

Each pollution source is given a basic quota e_i^0, and its total quota-permit is denoted as E^0. The total basic quota given by environmental protection authorities in the beginning of each fiscal year is not to exceed the standard of total emissions, meaning that E^0 stays smaller or equal to Q^s. Also one of the emission standard $q^s_{j'}$ assigned to each receptor is not to exceed the basic quota assigned to each pollution source.

Basing on the quota-permit can be tradable among the pollution sources, the QPS Model was formulated such that the total emissions amount finally can not over the standard of total emissions and the total reduction costs can be minimized. QPS Model can reach to the purpose of reducing the pollution emissions with minimal cost that is also the target of environmental protection authority (EPA) when s/he designs the policy.

3.2 Cost Effectiveness

In Model (2), $C_i(e_i)$ is of a twice continuously differential and convex in e_i, and its marginal reduction cost, $-C_i'(e_i)$, is of a positive and strictly increasing function. Therefore, if $C_i'(e_i)>0$, then we have $C_i''(e_i)>0$. Since each pollution source's anticipated pollution reduction cost + quota-permit cost + quota-permit operating fee being at a minimal level, thus $\frac{\partial z}{\partial e_i} = 0$, which derives,

$$C_i'(e_i) + P = 0, \text{ or } P = -C_i'(e_i) \qquad (3)$$

Thus as long as every pollution source does the reduction measure shown in equation (3), each pollution source's quota-permit cost, P, will equal to the marginal reduction cost, $-C_i'(e_i)>0$, to help attain an optimal cost effectiveness.

3.3 Implementation

Suppose that a total amount control region has reached its designated air quality level. A tradable quota-permit can be purchased from other pollution sources. Or EPA allocates a new quota-permit to a fixed pollution source. Providing that the amount of pollution emission sought by any new listing or modification submitted by fixed pollution sources would be tested through the model simulation indicating that it will not exceed the standard of total emissions amount assigned to a specific region. On the other hand, suppose that a total amount control region fails to reach its designated air quality level. Then, an overall reduction of the emission will be sought based on the target deadline in conjunction with the basic quota-permit assigned to all fixed pollution sources within the control region set by EPA, in order to achieve the anticipated quality standard. Environmental protection authorities would then collect the quota-permit fee, P_0, based on each annual budget forecast and the basic quota-permit released, $\sum_{i=1}^{k} e_i$ Nevertheless, at any given time the basic quota-permit released by environmental protection authorities is not to exceed the standards of total emission amount, nor that regulates each pollution source being the applicable pollution tax collectable by environmental protection authorities on pollutants emitted by a given pollution source.

For regions that receive the total emissions amount, the amount of emission is calculated based on per kilogram in order to tabulate the quota-permit, for the duration of one year. And after a trading system being finalized by environmental protection authorities, competitive bidding will be used to conduct a one-on-one trading, provided that no trading takes place between different categories of pollutants or between different control regions.

3.4 Limitation

The advantages of the QPS model lie in that not only environmental protection authorities may regulate the pollutant quality standard within a total amount control region through handing out the basic quota-permit, but it also provides a most cost efficient model for the entire system. And since quota-permit trading is only limited to within the same region, this will also help ensure that no trading takes place among different regions, thus help eliminate hot spots of congregated pollution sources.

As example, the greenhouse effect would only pose certain threats when the global emission of carbon dioxide (CO_2) reaches a certain level. Therefore, this allows the carbon dioxide gas quota-permit to be traded among different control regions, provided that none of the regional or national total emission standards were violated. The only shortcoming could be an artificially quota-permit price, P; suppose a majority of the basic quota-permit out on the market were manipulated by a small number of pollution sources, the quota-permit trading prices can easily be fixed. And unless it can be reasonably assume that such quota-permit market is of a completely market perfection, meaning none of the market power (Westskog, 1996) but consists of a great number of evenly competitive participants, the pollution resources, it may be difficult to maintain a total cost effectiveness. In addition, since environmental protection authorities may need to rely on the pollutant quality diffusion or transfer model to conduct the simulation in order to very whether the total emissions amount and environmental quality within a given region meet the nationwide pollutant quality control standards. Thus it is also assumed that the pollutant quality diffusion model adopted is capable of providing adequate simulation of anticipated results. In addition, such as that environmental protection authorities are of an efficient government, where the allocation and budgeting of the pollutant pollution surcharged levied to all fixed pollution sources are built around a rational, legal, fair basis. Or else, the disproportional surcharge pegged to the total amount emissions scheme on the quota-permit operating fee, $P_0 \sum_i e^0_i$, collected will only be crippled as an internal revenue deficiency that stymies the economy, increase the total control cost, $z(e_i)$, wasting the national resources.

Comparing the model between QPS and APS, we conclude as;

First, the QPS model rates the emission in its quota permit. Second, the trading is of a one-on-one basis. Third, it poises to achieve the d rule of cost efficiency. Forth, it is easy to implement, thus the quota-permit operating fees, $P_0\sum_i e_i^0$, remains low. Fifth, the quota-permit trading costs being identical to the shadow price. Sixth, it is easy to regulate since all pollutants are divided into different control regions. All of these come together to indicate that all advantages provided by the alternative plans support the hypothesis that this can be first be adopted to the carbon dioxide or any greenhouse gas emission reduction policy.

4. Conclusion

In order to reach the target of total amount control, we formulate a mathematical optimization model, QPS Model. The QPS Model is constructed under the tradable quota-permit policy used by a policymaker. The QPS Model can be used to analyze the issue of controlling pollutant emissions and environmental quality. APS can not reach to the requirements of pollutant quality standard, QPS can do.

This paper compared the differences between the APS and the QPS model. We suggest the QPS model is better one than APS, but it should set up the emission inventory before using QPS model in the real world. The QPS Model can conclude the optimal strategies to be decided by a policymaker of environmental protection authorities. In order to approve the power of QPS model, it should solve the model (2) and experiment with actual numeric data as our future studies.

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